For full credit please show your work and write all proofs using complete sentences. No wireless devices are permitted.

1. Show that the set $A$ given as follows is bounded above. Prove that $\sup A = 2$ using the definition.

$$A = \left\{ 2 - \frac{1}{n+1} : n \in \mathbb{N} \right\}$$

For $n \in \mathbb{N}$, $\frac{1}{n+1} > 0$; so $2 - \frac{1}{n+1} \leq 2$. Hence 2 is an upper bound for $A$.

Let $x < 2$. Then $2 - x > 0$ so $\exists n \in \mathbb{N} \text{ s.t. } \frac{1}{n} < 2 - x$. Hence $\frac{1}{n+1} < \frac{1}{n} < 2 - x$ and so $x < 2 - \frac{1}{n+1}$.

Since $2 - \frac{1}{n+1} \in A$, $x$ is not an upper bound for $A$. Thus 2 is the least upper bound for $A$ and hence $\sup A = 2$. 