For full credit please show your work and write all proofs using complete sentences. No wireless devices are permitted.

1. Let $f$ and $g$ be two continuous functions defined on $I = [a, b]$ such that $f(a) < g(a)$ and $g(b) < f(b)$. Prove that there exists $c \in I$ such that $f(c) = g(c)$.

   Let $h : I \to \mathbb{R}$ be given by $h(x) = f(x) - g(x)$ for $x \in I$. Then $h$ is continuous because it is the difference of two continuous functions. Since $f(a) < g(a)$ and $f(b) > g(b)$, $h(a) = f(a) - g(a) < 0$ and $h(b) = f(b) - g(b) > 0$. Thus $h(a) < 0 < h(b)$ and so by the IVT $h(c) = 0$ for some $c \in I$. Since $f(c) - g(c) = h(c) = 0$, $f(c) = g(c)$. 