For full credit please show your work and write all proofs using complete sentences. No wireless devices are permitted.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$ f(x) = \begin{cases} x^2 & \text{if } x \leq 1, \\ \sqrt{x} & \text{if } x > 1. \end{cases} $$

Determine whether $\lim_{x \to 1} f(x)$ exists. Is $f$ continuous at 1? Explain.

Since the function $x \mapsto \sqrt{x}$ is continuous at 1 we have,

$$ \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \sqrt{x} = \sqrt{1} = 1. $$

Since the function $x \mapsto x^2$ is also continuous at 1 we have

$$ \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1^2 = 1. $$

Observe that $\lim_{x \to 1^+} f(x) = 1 = \lim_{x \to 1^-} f(x)$. Hence $\lim_{x \to 1} f(x)$ exists and $\lim_{x \to 1} f(x) = 1$. Since $f(1) = 1 = \lim_{x \to 1} f(x)$, $f$ is continuous at 1.