1. Prove that \( n^{1/n} \to 1 \). (Hint: First show \( \lim_{x \to \infty} x^{1/x} = 1 \).)

2. Let \( \{a_n\}_n \) be a nonnegative sequence, let \( \{b_n\}_n \) be a bounded sequence and suppose that \( \sum_{n=1}^{\infty} a_n \) converges. Prove that \( \sum_{n=1}^{\infty} a_n b_n \) converges. Does the series converge absolutely?

3. Let \( \sum_{n=1}^{\infty} a_n \) and \( \sum_{n=1}^{\infty} b_n \) be series and suppose that \( b_n > 0 \) for all \( n \geq 1 \) and \( \frac{a_n}{b_n} \to 0 \). Prove that if \( \sum_{n=1}^{\infty} b_n \) converges, then \( \sum_{n=1}^{\infty} a_n \) converges absolutely.

4. Let \( \sum_{n=1}^{\infty} a_n \) be a series with nonzero terms and suppose that \( \limsup \frac{|a_{n+1}|}{|a_n|} < 1 \). Prove that \( \sum_{n=1}^{\infty} a_n \) converges absolutely.

5. In each case determine whether the given series converges. Prove your answer and cite any test you use by name.
   a. \( \sum_{n=10}^{\infty} \frac{(\ln n)^2}{n} \)
   b. \( \sum_{n=1}^{\infty} \frac{n^2}{((-1)^n + 3)^n} \)