Please do the problems in order, write clearly and justify your answers. Please use complete sentences.

1. Let $E = \{(x, y) \in \mathbb{R}^2 : 1 < x^2 + y^2 < 4\}$. Show that $E$ is open. Find $E^o$, $E$ and $\partial E$.

2. Let $E \subset \mathbb{R}^d$ be nonempty.
   
   (a) Prove the second assertion in 7.3i(vii): $(E^o)^c = \overline{E^c}$.
   
   (b) Let $E \subset \mathbb{R}^d$. Use the above to prove 7.3i(iv): $\partial E = \overline{E} \cap \overline{E^c}$.

3. Let $A$ and $B$ be subsets of $\mathbb{R}^d$.
   
   (a) Prove that $(A \cap B)^o = A^o \cap B^o$.
   
   (b) Either prove that $(A \cup B)^o = A^o \cup B^o$ or find a counterexample.

4. Use the definition of compactness (7.4c or [7.4.3]) to prove that every finite set is compact.

5. Let $K \subset \mathbb{R}^d$ be a nonempty compact set. Why $M = \sup_{x \in K} \|x\|$ is finite? Prove that $M = \|x\|$ for some $x \in K$.
   
   Hint: If $K = \{0\}$ the result is clear. If not, suppose there is no such $x$ and then show that $\mathcal{U} = \{B_r(0) : 0 < r < M\}$ is an open cover of $K$. 
