Let $W = \{(x,y) \in \mathbb{R}^2 : -2 \leq 2x+y \leq 2, 0 \leq 2x-y \leq 6\}$

Eval \[\int_W (4x^2 - y^2) \, dV(x,y)\]

Observe that $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ def by $\phi(x,y) = (2x+y, 2x-y)$ for $(x,y) \in \mathbb{R}^2$ and observe that $[d\phi(x,y)] = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix}$ with $\det [d\phi(x,y)] = -4 \neq 0$ for all $(x,y) \in \mathbb{R}$. Note that $\phi$ is bijective and thus 1-1 on $W$. We have $\phi(W) = [-2, 2] \times [0, 6]$

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(u,v) = \frac{1}{4} u^2 v^2$, then $f$ is cts and so int. on $\phi(W)$. And since $W, \phi(W)$ are Jordan, 10.5b applies and so by the Change of Variables Formula:

\[\int_{\phi(W)} \frac{1}{4} u^2 v^2 \, dV(u,v) = \int_W \frac{1}{4} (2x+y)^2 (2x-y)^2 + 4 \, dV(x,y) = \int_W (4x^2 - y^2) \, dV(x,y)\]

Moreover by 10.4f, we have

\[
\int_{\phi(W)} \frac{1}{4} u^2 v^2 \, dV(u,v) = \frac{1}{4} \int_{-2}^{2} \left[ \int_{0}^{6} u^2 v^2 \, dv \right] du = \frac{1}{4} \int_{-2}^{2} u^2 \left[ \frac{v^3}{3} \right]_{0}^{6} \, du = \frac{1}{4} \int_{-2}^{2} u^2 \left[ \frac{216}{3} \right] \, du = 18 \int_{-2}^{2} u^2 \, du = 6u^3 \bigg|_{-2}^{2} = 96
\]