For full credit please show your work and write all proofs using complete sentences. No wireless devices are permitted.

1. Determine whether the given series converges. Prove your answer and cite any test you use by name.

\[ \sum_{n=1}^{\infty} \frac{n^3}{3^n} \]

One can either use the Root Test or the Ratio Test to prove that the given series converges.

Observe that

\[ |a_n|^{1/n} = \left| \frac{n^3}{3^n} \right| = \frac{(n^3)^{1/n}}{3} \rightarrow \frac{1}{3} = \frac{1}{3} \, . \]

Hence, \( \limsup |a_n|^{1/n} = \frac{1}{3} < 1 \) and so by the Root Test, the series converges.

Alternatively, we have

\[ \left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3} \right| = \frac{1}{3} \left( \frac{n+1}{n} \right)^3 = \frac{1}{3} \left( 1 + \frac{1}{n} \right)^3 \rightarrow \frac{1}{3} \, . \]

Since \( \lim \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{3} < 1 \), the series converges by the Ratio Test.