For full credit please show your work and write all proofs using complete sentences. No wireless devices are permitted.

1. For \( K \subset \mathbb{R}^d \), define \( \text{diam} \, K = \sup \{ \| x - y \| : x, y \in K \} \). Prove that \( \text{diam} \, K < \infty \), if \( K \) is compact.

Sps that \( K \) is compact. Then by the Heine-Borel Theorem \( K \) is bounded and thus \( \exists M > 0 \) st. \( \| x \| \leq M \) for all \( x \in K \).

Let \( x, y \in K \), then \( \| x - y \| \leq \| x \| + \| y \| \leq 2M \) and hence the set \( S = \{ \| x - y \| : x, y \in K \} \) is bounded above. Therefore, \( \sup S < \infty \).