1.2. Let \( S = \{-2, -1, 0, 1, 2, 3\} \). Describe each of the following sets as \( \{x \in S : p(x)\} \), where \( p(x) \) is some condition on \( x \).

(b) \( B = \{0, 1, 2, 3\} \)

Note that \( B \) consists of all the nonnegative elements of \( S \). Hence, we have \( B = \{x \in S : x \geq 0\} \). □

(d) \( D = \{-2, 2, 3\} \)

An element \( x \in S \) is an element in \( D \) exactly when \( |x| \geq 2 \). Hence, we have \( D = \{x \in S : |x| \geq 2\} \). □

1.4. Write each of the following sets by listing its elements within braces.

(b) \( B = \{n \in \mathbb{Z} : n^2 < 5\} \)

Note that an integer \( n \in \mathbb{Z} \) is an element of \( B \) exactly when \( |n| < \sqrt{5} \), that is, \( |n| \leq 2 \). Therefore, \( B = \{-2, -1, 0, 1, 2\} \).

(d) \( D = \{x \in \mathbb{R} : x^2 - x = 0\} \)

The equation \( x^2 - x = 0 \) has two real solutions: \( x = 0, 1 \). So \( D = \{0, 1\} \).

1.6(c) The set \( E = \{2x : x \in \mathbb{Z}\} \) can be described by listing its elements, namely \( E = \{\ldots, -4, -2, 0, 2, 4, \ldots\} \).

List the elements of the following set in a similar manner.

\( C = \{3q + 1 : q \in \mathbb{Z}\} \)

Setting \( q = -2, -1, 0, 1, 2 \) we obtain \( 3q + 1 = -5, -2, 1, 4, 7 \). This establishes a clear pattern and we have \( C = \{\ldots, -8, -5, -2, 1, 4, 7, 10, \ldots\} \).

1.10 Which of the following sets are equal?

\[
\begin{align*}
A &= \{n \in \mathbb{Z} : |n| < 2\} \\
B &= \{n \in \mathbb{Z} : n^3 = n\} \\
C &= \{n \in \mathbb{Z} : n^2 \leq n\} \\
D &= \{n \in \mathbb{Z} : n^2 \leq 1\} \\
E &= \{-1, 0, 1\}
\end{align*}
\]

All of the above sets with exception of \( C \) are equal. Let \( n \) be an integer. Then \( n \in A \) exactly when \( |n| \leq 1 \), that is, when \( n = -1, 0, 1 \). Similarly, \( n^3 = n \) precisely when \( n = -1, 0, 1 \); and \( n^2 \leq 1 \) exactly when \( |n| \leq 1 \), i.e., when \( n = -1, 0, 1 \). But \( n^2 \leq n \) fails to hold for \( n = -1 \) and only holds for \( n = 0, 1 \). Hence, \( A = B = D = E \) but \( C = \{0, 1\} \neq E \).

1.14 Find \( \mathcal{P}(\mathcal{P}\{\{1\}\}) \) and its cardinality.

We have

\[
\begin{align*}
\mathcal{P}\{\{1\}\} &= \{\emptyset, \{1\}\}, \\
\mathcal{P}(\mathcal{P}\{\{1\}\}) &= \mathcal{P}(\{\emptyset, \{1\}\}) \\
&= \{\emptyset, \{\emptyset\}, \{\{1\}\}, \emptyset, \{1\}\}.
\end{align*}
\]

Since \( \mathcal{P}(\mathcal{P}\{\{1\}\}) \) has four elements, we have \( |\mathcal{P}(\mathcal{P}\{\{1\}\})| = 4 \). □

1.18(b) Give examples of three sets \( A, B \) and \( C \) such that \( B \in A, B \subseteq C \) and \( A \cap C \neq \emptyset \).

There are many possible solutions. Perhaps the simplest is given by \( B = \emptyset \) and \( A = C = \{\emptyset\} \). Then \( B \) is certainly an element of \( A \) as well as a proper subset of \( C \). Moreover, \( A \cap C = \{\emptyset\} \neq \emptyset \).

Alternatively, take \( A = \{\{1\}\}, B = \{1\}, C = \{1, \{1\}\} \). Then \( B \subset C \) and since \( A = \{B\} \), \( B \in A \).

Moreover, \( A \subseteq C \), so \( A \cap C = A \neq \emptyset \). □
1.22 Give an example of a universal set $U$, two sets $A$ and $B$, and an accompanying Venn diagram such that $|A \cap B| = |A - B| = |B - A| = |A \cup B| = 2$.
Let
$$U = \{s, t, u, v, w, x, y, z\},$$
$$A = \{s, t, u, v\},$$
$$B = \{u, v, w, x\}.$$
Then
$$A \cap B = \{u, v\},$$
$$A - B = \{s, t\},$$
$$B - A = \{w, x\},$$
$$A \cup B = \{y, z\}.$$
Each of these sets has two elements.

1.28 For a real number $r$, define $S_r$ to be the interval $[r - 1, r + 2]$. Let $A = \{1, 3, 4\}$. Determine $\bigcup_{\alpha \in A} S_\alpha$ and $\bigcap_{\alpha \in A} S_\alpha$.
Note that $S_1 = [0, 3]$, $S_3 = [2, 5]$ and $S_4 = [3, 6]$ and so
$$\bigcup_{\alpha \in A} S_\alpha = S_1 \cup S_3 \cup S_4 = [0, 3] \cup [2, 5] \cup [3, 6] = [0, 6],$$
$$\bigcap_{\alpha \in A} S_\alpha = S_1 \cap S_3 \cap S_4 = [0, 3] \cap [2, 5] \cap [3, 6] = \{3\}.$$