MATH 352, FALL, PRACTICE EXERCISES FOR TEST 2

INSTRUCTIONS: Please try and do these problems without looking at the book. Solutions are provided at the end of this document. Your test questions will differ from this practice questions. You may need the normal and t-distribution tables for the exam - make sure you have copies with you! All questions that appear below, and those on the test, may be asked as open-ended or multiple choice questions.

ENJOY!

1. True or false: Circle the correct answer.
T  or F. The Type I error in testing hypotheses: Ho: μ=2 vs. H1: μ≠ 2 is the probability that we conclude μ≠ 2 given that in reality μ=2.

T or F. If a null hypothesis is rejected at the 0.05 significance level, does it necessarily have to be rejected at the 0.01 significance level?

T or F. A 99% confidence interval for the population mean \( \mu \) is determined to be (65.32 to 73.54). If the confidence level is reduced to 90%, the 90% confidence interval for \( \mu \) becomes narrower.

2. An electrical firm that manufactures a certain type of bulb wants to estimate its mean life. Assuming that the life of the bulbs has a normal distribution with a standard deviation \( \sigma = 40 \) hours, find how many bulbs should be tested so as to be 90 percent confident that the sample mean will not differ from the true mean life by more than 10 hours.

3. Let \( p \) denote the proportion of all potential subscribers who favor cable company A over cable company B. Consider testing \( H_0: p \leq 0.5 \) versus \( H_1: p > 0.5 \) based on a random sample of 9 individuals. Suppose that the null hypothesis is rejected if \( X \geq 8 \), where \( X \) is the number of individuals in the sample who favor company A.

(a) Describe what Type I and Type II errors are in the context of this problem.

(b) What is the probability distribution of the test statistic \( X \) when \( H_0 \) is true and \( p=0.5 \)? Use it to derive the probability of Type I error.

(c) What would you conclude if 5 of the 9 individuals in the sample favored company B?

4. Let \( \mu \) denote the true mean tread life of a certain type of tire. Consider a level \( \alpha = 0.05 \) test of \( H_0: \mu \leq 20,000 \) versus \( H_1: \mu > 20,000 \) based on a sample of size \( n=16 \) from a normal population with \( \sigma = 1500 \). Carry out the test if \( \bar{X} = 20,960 \). What is the probability of Type I error?

5. Suppose that the weight of golf balls is normally distributed with mean of 9.5 grams and standard deviation of 1.2 grams. I pack 36 golf balls into a bag. Find the probability that the bag will weigh less than 360 grams? (Hint: Use the Central Limit Theorem)

6. A CI is desired for the true average stray-load loss \( \mu \) (watts) for a certain type of induction motor when the line current is held at 10 amps for a speed of 1500 rpm. Assume that stray-load loss is normally distributed. Compute a 95% CI for \( \mu \) when \( n = 25 \), \( s = 3.0 \), and \( \bar{x} = 60 \).
7. A random sample of 100 lightning flashes in a certain region resulted in a sample average radar echo duration of .81 sec and a sample standard deviation of .34 sec. Calculate a 99% confidence interval for the true average echo duration \( \mu \), and interpret the resulting interval.

8. A random sample of \( n = 8 \) E-glass fiber test specimens of a certain type yielded a sample mean interfacial shear yield stress of 30.5 and a sample standard deviation of 3.0. Assuming that interfacial shear yield stress is normally distributed, compute a 95% CI for true average stress.

9. In their annual report, the State Department of Education stated that 62% of academic employees in the State were permanent faculty and the remaining 38% were temporary. A professor in Bumbleton State University (BSU) claims that BSU has a lower proportion of permanent faculty than the overall State proportion of 62%. A random sample of 400 academic employees at BSU showed 215 of them to have permanent positions.

a. Find a 90% confidence interval for the true proportion of permanent academic employees at BSU.

b. How large a sample is required in order to be 99% confident that the sample proportion will not differ from the true proportion of permanent faculty at BSU by more than 0.02. Use the sample information stated in the problem for your initial estimate of \( p \).

c. Assume that no prior information about the proportion of permanent faculty at BSU is available. How large a sample is required to be 99% confident that the sample proportion of permanent faculty at BSU will not differ from the true proportion by more than 0.02.

d. Test the professor’s claim at 0.01 significance level.

10. The people representative claims that the true mean medical expenses during a year (for a family) are greater than $750. In a survey in which 100 randomly chosen middle-class families were interviewed, it was found that their mean medical expenses during a year were $770 with standard deviation of $120. Is the rep’s claim justified? Test appropriate hypothesis using a significance level \( \alpha = 0.025 \).

11. A sample of 12 radon detectors of a certain type was selected, and each was exposed to 100 pCi/L of radon. The resulting readings were as follows:

\[
\begin{array}{cccccc}
104.3 & 89.6 & 89.9 & 95.6 & 95.2 & 90.0 \\
98.8 & 103.7 & 98.3 & 106.4 & 102.0 & 91.1 \\
\end{array}
\]

Does this data suggest that the population mean reading under these conditions differs from 100? State and test the appropriate hypotheses using significance level \( \alpha = .05 \).

12. State DMV records indicate that of all vehicles undergoing emissions testing during the previous year, 70% passed on the first try. A random sample of 200 cars tested in a particular county during the current year yields 160 that passed on the initial test. Does this suggest that the true proportion for this county during the current year differs from the previous statewide proportion? Test the relevant hypotheses using \( \alpha = 0.05 \).

13. Tensile strength tests were carried out on two different grades of wire rod resulting in the accompanying data:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Sample size</th>
<th>Sample mean</th>
<th>Sample st. deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI 1064</td>
<td>130</td>
<td>108</td>
<td>1.3</td>
</tr>
<tr>
<td>AISI 1078</td>
<td>130</td>
<td>124</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Estimate the difference between true average strengths for the two grades using a 90% confidence interval.

14. To decide whether two different types of steel have the same true average fracture toughness a test was performed on samples from both types. A summary data on proportional stress limits for specimens constructed using two different types of wood are shown below:

<table>
<thead>
<tr>
<th>Type of wood</th>
<th>Sample size</th>
<th>Sample mean</th>
<th>Sample St.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red oak</td>
<td>14</td>
<td>8.50</td>
<td>.80</td>
</tr>
<tr>
<td>Douglas fir</td>
<td>10</td>
<td>6.65</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Assuming that both samples were selected from normal distributions, find a 95% CI for the difference in average proportional stress limit for red oak joints and the Douglas fir joints.

15. A breeder of rabbits claims that he can breed rabbits yielding a mean weight of greater than 58 ounces. A random sample of 16 rabbits had a mean weight of 59.2 ounces and standard deviation of weights $s = 3$ ounces. Assume normal distribution of rabbits’ weights.
   a. Is the breeder’s claim justified? Use 5% level of significance.
   b. Find a 95% confidence interval for the true mean weight of a rabbit.

16. A brewery distributes beer in cans labeled 12 oz. The Bureau of Weights and Measures randomly selects 36 cans, measures their mean contents, and finds their mean weight to be 11.82 oz. Assume that $\sigma$ is known to be 0.38 oz.
   a. Find a 95% confidence interval for the true mean content of a can.
   b. How large a sample is required in order to be 95% confident that the sample mean will not differ from the true mean content by more than 0.01.

17. Ten voters were picked at random from those that voted in favor of a proposition, and 12 voters were picked at random from those who voted against it. Ages of the voters were recorded. The mean age of the “in favor” group was 38 years with a standard deviation $s=8$ years. The mean age of the “against” group was 38 years with a standard deviation of 7 years. Find a 95% CI for the true difference in age between those voting against the proposition and those voting for it? Assume equal variances of ages of the two populations of voters.

18. A medical study was looking into the question if using a drug to reduce blood cholesterol level will decrease the risk of a heart attack. Middle age men were randomly assigned to two groups. One group of 2051 men took a drug that reduces cholesterol level, and the second group (control) of 2030 men took a placebo. During the next five years, 56 men from the treatment group and 84 men from the control group had heart attacks. Find a 95% CI for the difference in proportions of those who had a heart attack after using the drug and those who had a heart attack not using the drug.

19. The time $X$ (minutes) for a lab assistant to prepare the equipment for a certain experiment is believed to have a lognormal distribution with parameters $\mu = 1.2$ hrs and $\sigma = 0.4$ hrs.
   a. What is the mean prep time?
   b. What is the probability that preparation time exceeds 3 hrs?
   c. What is the 90th percentile of the prep times?
ANSWERS

Problem 1. T, F, T.

Problem 2. Here \( \sigma=10 \), \( m=10 \), \( \alpha=0.1 \), so \( \alpha/2=0.05 \). Thus \( n = \left( \frac{z_{\alpha/2} \sigma}{m} \right)^2 = \left( \frac{1.645 (40)}{10} \right)^2 = 43.3 \), round up \( n=44 \).

Problem 3.

a. Type I error = reject Ho when it is true= decide that over 50% of potential subscribers prefer Company A, when in fact at most 50% of subscribers prefer A.

Type II error= accept Ho when it is false= decide that at most 50% of potential subscribers prefer Company A, when in fact more than 50% of subscribers prefer A.

b. The test statistic in this problem is \( X = \) number of individuals in the sample of 9 who prefer A. When Ho is true and \( p=0.5 \), then the distribution of \( X \) is Binomial(9, 0.5).

The decision rule given in the problem reject Ho when \( X \geq 8 \). Thus,
\[
P(\text{Type I error}) = P(X \geq 8 \text{ given } X \text{ is Binomial}(9, 0.5)) = 1 - P(X \leq 7) = \text{MINITAB}= 1-0.9805= 0.0195.
\]

Cumulative Distribution Function

<table>
<thead>
<tr>
<th>Binomial with n = 9 and p = 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>

c. If \( X=5 \), then using the decision rule described in the problem, we would not reject Ho, and conclude that the proportion of potential subscribers who prefer Company A does not exceed 0.5.

Problem 4. \( \mu= \) true mean thread life of a tire, \( n=16 \), \( \sigma=1500 \), \( \bar{X} = 20,960 \), normal population of lifetimes. Test hypotheses:

Ho: \( \mu \leq 20,000 \) versus \( H_1: \mu > 20,000 \)

Test statistic \( z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{20,960 - 20,000}{1500/\sqrt{16}} = 2.56 \) (I used MINITAB)

Critical number: \( z(0.05) = 1.645 \), Decision rule: reject Ho when the test statistic \( > 1.645 \)

\[
P(\text{Type I error}) = P(\text{reject Ho when Ho true}) = P( \text{test statistic} > 1.645 \text{ given } \mu=20,000) = P(Z>1.645 \text{ given } Z \text{ is standard normal}) = 0.05.
\]

One-Sample Z

Test of \( \mu = 20000 \) vs not = 20000
The assumed standard deviation = 1500

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>SE Mean</th>
<th>95% CI</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>20960</td>
<td>375</td>
<td>(20225, 21695)</td>
<td>2.56</td>
<td>0.010</td>
</tr>
</tbody>
</table>
Problem 5. Weights of golf balls have a normal distribution with mean=9.5, and standard deviation =1.2 grams. Then, the sample mean of 36 balls $\bar{X} \sim N \left(9.5, \frac{1.2^2}{36}\right) = N(9.5, 0.04)$. Thus $P(\text{total weight } < 360) = P(\frac{\bar{X} < 360}{36}) = P(\bar{X} < 10) = P(Z < (10 - 9.5)/ 0.2) = P(Z < 2.5) = 0.9938$

Problem 6. $\mu =$ mean stray-loss (watts), normal population of stray losses.

95% CI for $\mu$: $\bar{X} \pm t_{n-1,0.025} \frac{s}{\sqrt{n}} = I$ used MINITAB = (58.762, 61.238). If you were using the formula and calculator, then use $t(0.025, 24) = 2.064$

One-Sample T

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>60.00</td>
<td>3.000</td>
<td>0.600</td>
<td>(58.762, 61.238)</td>
</tr>
</tbody>
</table>

Problem 7. 99% CI for $\mu=$ mean length of radar echo:

$\bar{X} \pm t_{n-1,0.005} \frac{s}{\sqrt{n}} = I$ used MINITAB = (0.7207, 0.8993). If I needed to use the t-table, I need $t$ with 99 df which is not listed. I would have used $t$ distribution with df=120 which is available and closest to 99 df. I would get $t(0.005) = 2.617$. The CI would be a little different than the one done in MINITAB. The reason is that MINITAB uses exact value of $t(0.005, 0.005)$. We could also use z-percentile in place of t-percentile, because the sample size is large.

One-Sample T

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>99% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.8100</td>
<td>0.3400</td>
<td>0.0340</td>
<td>(0.7207, 0.8993)</td>
</tr>
</tbody>
</table>

Problem 8. 95% CI for $\mu=$ mean stress:

d.f. = $n-1 = 7$, so the critical value for a 95% C.I. is $t_{0.05, 7} = 2.365 = 2.365$. The interval is then $30.5 \pm 2.365 \left(\frac{3.0}{\sqrt{8}}\right) = 30.5 \pm 2.51(27.99, 33.01)$.

Problem 9.

a. Let $p=$ the true proportion of permanent academic employees at BSU. We need a 90% confidence interval for $p$.

I used the traditional formula: $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, with $\hat{p} = 215/400 = 0.5375$, $n = 400$, $z_{0.025} = 1.645$, so 90% CI for $p$ is: (0.4965 , 0.5785)

IN MINITAB: result a bit different due to rounding
Test and CI for One Proportion

<table>
<thead>
<tr>
<th>Sample</th>
<th>X</th>
<th>N</th>
<th>Sample p</th>
<th>90% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>215</td>
<td>400</td>
<td>0.537500</td>
<td>(0.495138, 0.579447)</td>
</tr>
</tbody>
</table>
b. How large a sample is required in order to be 99% confident that the sample proportion will not differ from the true proportion of permanent faculty at BSU by more than 0.02. Use the sample information stated in the problem for your initial estimate of \( p \).

I used the traditional formula:

\[
 n = \frac{z_{\alpha/2}^2 \hat{p}(1-\hat{p})}{m^2}
\]

with \( \hat{p} = 0.5375 \), \( z_{\alpha/2} = 2.576 \), and \( m = 0.02 \), we get \( n = 4125 \)

c. Assume that no prior information about the proportion of permanent faculty at BSU is available. How large a sample is required to be 99% confident that the sample proportion of permanent faculty at BSU will not differ from the true proportion by more than 0.02.

Since no prior information for \( p_0 \), we use \( \hat{p} = \frac{1}{2} \):

\[
 n = \frac{2.576^2(0.5)(1-0.5)}{0.02^2} = 4147.36 \approx 4148
\]

d. Test the professor’s claim at 0.01 significance level.

Step 1. \( H_0: \ p = 0.62 \) \( H_A: p < 0.62 \) (professor’s claim)

Step 2: value of the test statistics = \( z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = -3.39935 \)

Step 3: Critical value(s) = -2.326

Step 4: Decision: Since the test statistic \( z = -3.399 < -2.326 = \) critical number, then we reject \( H_0 \).

Step 5: Is the professor’s claim justified? Yes

Problem 10.

Let \( \mu = \) true mean family medical expenses (in $), \( n=100 \), \( \bar{X} = 770 \), and \( s = 120 \). Claim: the mean medical expenses are greater than 750. Significance level = 0.025.

Step 1. \( H_0: \ \mu \leq 750 \) \( H_A: \mu > 750 \) (claim of the people rep)

Step 2: value of the test statistics = \( t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = 1.67 \)

Step 3: Critical value comes from t distribution with 99 df. Since we do not have 99 df in the table, I used 120 df and thus \( t(0.025) = 1.98 \).

Step 4: Decision: Since the test statistic \( z = 1.67 < 1.98 = \) critical number, then we do not reject \( H_0 \).

Step 5: Is the people rep’s claim justified? No
**Problem 11. Radon detectors**

Let $\mu =$ true mean reading for exposure to 100 pCi/L. $n=12$, $\bar{X} = 97.07$, and $s=6.11$. Claim: the mean reading is different from 100. Significance level = 0.05.

Step 1. $H_0: \mu = 100$  \qquad $H_A: \mu \neq 100$ (mean reading differs from 100)

Step 2: value of the test statistics $= t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = -1.66$

Step 3: Critical values come from t distribution with 11 df, $t(0.025) = 2.201$, critical numbers: $\pm 2.201$.

Step 4: Decision: Since the test statistic $-2.201 < t= -1.66 < 2.201$, then we do not reject $H_0$.

Step 5: Is the mean reading different from 100? No, the data do not indicate that mean reading differs significantly from 100.

**One-Sample T: reading**

Test of mu = 100 vs not = 100

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95% CI</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>reading</td>
<td>12</td>
<td>97.07</td>
<td>6.11</td>
<td>1.76</td>
<td>(93.19, 100.96)</td>
<td>-1.66</td>
<td>0.125</td>
</tr>
</tbody>
</table>

**Problem 12. DMV problem**

Parameter of interest: $p =$ true proportion of cars in this particular county passing emissions testing on the first try. The sample proportion is $\hat{p} = 160 / 200 = 0.80$, significance level is 0.05.

$H_0 : p = 0.70$, and $H_A : p \neq 0.70$

The test statistic value is $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = 3.086$

Critical numbers: $z(0.025)=1.96$, so critical numbers: $\pm 1.96$.
Since the test statistic $= 3.086 > 1.96$, we reject $H_0$.
The data indicates that the proportion of cars passing the first time on emission testing or this county differs from the proportion of cars passing statewide.

**Test and CI for One Proportion**

Test of $p = 0.7$ vs $p$ not = 0.7

<table>
<thead>
<tr>
<th>Sample</th>
<th>X</th>
<th>N</th>
<th>Sample p</th>
<th>95% CI</th>
<th>Exact P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160</td>
<td>200</td>
<td>0.800000</td>
<td>(0.737774, 0.853106)</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Problem 13. tensile strength

Need 90% CI for $\mu_1 - \mu_2$, where $\mu_1 (\mu_2) =$true mean tensile strength for grade AISI1064(AISI 1078) wires. Large samples, so we use CI based on z statistic.

$$\bar{X} - \bar{Y} \pm z(0.05) \sqrt{\frac{s^2_X}{n_X} + \frac{s^2_Y}{n_Y}}$$

The $Z(0.05) = 1.645$, and the 90% CI is: (-16.345, -15.6546).
If I used MINITAB, I would have to ask for 2 sample t, because we do not know population standard deviations, but for such large samples, the value of the t-percentile, would be almost the same as the z-percentile. This is what MINITAB gives (note that the CI is almost the same as what I got above):

Two-Sample T-Test and CI

<table>
<thead>
<tr>
<th>Sample</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>130</td>
<td>108.00</td>
<td>1.30</td>
<td>0.11</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>124.00</td>
<td>2.00</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Problem 13. Wood problem

Let $\mu_1 =$ the true average proportional stress limit for red oak and let $\mu_2 =$ the true average proportional stress limit for Douglas fir.

We need 95% CI for $\mu_1 - \mu_2$. The sample sizes are small, so we need to use t-based CI. The number of df for t distribution will be

$$n = \frac{\left(\frac{s^2_X + s^2_Y}{n_X \cdot n_Y}\right)^2}{\left(\frac{s^2_X}{n_X - 1}\right)^2 + \left(\frac{s^2_Y}{n_Y - 1}\right)^2} = 14.13$$

So, we use 14 df. Thus, $t(0.025,14) = 2.145$, and the 95% CI is:

$$\bar{X} - \bar{Y} \pm 2.145 \sqrt{\frac{s^2_X}{n_X} + \frac{s^2_Y}{n_Y}} = (0.876, 2.824)$$

MINITAB gives: (note similar CI, difference due to rounding)

Two-Sample T-Test and CI

<table>
<thead>
<tr>
<th>Sample</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>8.500</td>
<td>0.800</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>6.65</td>
<td>1.28</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Difference = $\mu_1 (1) - \mu_2 (2)$
Estimate for difference: 1.850
95% CI for difference: (0.861, 2.839)
T-Test of difference = 0 (vs not =): T-Value = 4.04  P-Value = 0.001  DF = 13
**Problem 15. Rabbit problem;**
Let $\mu =$mean weight of rabbit

a. Step 1. $H_0$: $\mu \leq 58$ \quad $H_A$: $\mu > 58$ (breeder’s claim)

Step 2:
Test statistic: $t_0 = (\bar{x} - 58)/(s/\sqrt{n})=(59.2 -58)/(3/4)=1.6$

Step 3: Critical value(s) = 1.75

Critical value = $t(n-1), \alpha = t(15,.05) = 1.75$

Step 4: Decision: Do not reject $H_0$.

Step 5: Is the breeder’s claim justified? **No,** the breeder’s claim for $\mu >58$ is NOT justified !

b. The 95% confidence interval for $\mu$ is

$$[\bar{x} - t(n-1), \alpha/2 \cdot (s/\sqrt{n}) , \bar{x} + t(n-1), \alpha/2 \cdot (s/\sqrt{n}) ]$$

$$= [59.2 - t(15,.025),(3/4) , 59.2 + t(15,.025),(3/4) ]$$

$$= [59.2 -2.131*.75 , 59.2 +2.131*.75 ]$$

$$= [57.60 , 60.79 ]$$

**Problem 16. Brewery problem.**

a. let $\mu =$ true mean content of a can. The 95%CI for $\mu$ is:

$$(\bar{X} \pm Z\alpha/2 \cdot \frac{\sigma}{\sqrt{n}}) = (11.82 \pm (1.96)(0.38/\sqrt{36})) = (11.696,11.944)$$

b. The sample size is:

$$n = (z(alpha/2)*(sigma/m))^2$$

$$= ((-1.96)(0.38)/(0.01))^2$$

$$= 5547.2704$$

Hence, we would need a sample size of $n = 5548$ to be 95% confident that the sample mean will not differ from the true mean content by more than 0.01 (margin of error).

**Problem 17.**
Let $\mu(x) =$ mean age of voters in favor of the prop, and $\mu(y) =$ mean age of voters against the prop.

Given $m = 10$, $\bar{x} = 38$, $S_x = 8$; $n = 12$, $\bar{y} = 38$, $S_y = 7$. Need 95% CI for $\mu(x) - \mu(y)$. We assume equal variances of ages of the two populations of voters. Need pooled estimate of common variance:

$$S_p^2 = \frac{(m-1)S_x^2 + (n-1)S_y^2}{m+n-2} = \frac{(10-1)8^2 + (12-1)7^2}{8+7-2} = 85.7692$$

The 95% CI is:

$$\bar{X} - \bar{Y} \pm t_{n_x+n_y-2,\alpha/2} \cdot S_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}.$$

So, the 95% CI is (-6.67, 6.67)
Problem 18. Heart problem

px = proportion of suffering heart attacks in treatment group
py = proportion of suffering heart attacks in control group;
Sample statistics: \( \hat{p}_x = 0.0278, \hat{p}_y = 0.042 \). The formula for the CI is

\[
\hat{p}_x - \hat{p}_y \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_x (1 - \hat{p}_x)}{\hat{n}_x} + \frac{\hat{p}_y (1 - \hat{p}_y)}{\hat{n}_y}}
\]

The CI is (-0.0255, -0.0029).

Problem 19. Let \( X \) = prep time in hours, \( X \sim \text{lognormal}(1.2, 0.4^2) \). Let \( Y = \ln X \sim \text{N}(1.2, 0.4^2) \)

a. \( \text{EX} = e^{\mu + \sigma^2/2} = e^{1.2 + 0.4^2/2} = 3.597 \). The mean prep time is 3.6 hrs or 3 hrs and 36 min.

b. \( P(X > 3) = P(\ln X > \ln 3) = P(Y > 1.1) = P(Z > -0.25) = 0.5987. \)

c. Let \( x(90) = 90^{th} \) percentile of the waiting times. That is \( P(X > x(90)) = 0.1 \). Thus, \( P(\ln X > \ln x(90)) = 0.1 \), standardizing we get:

\( P(Z > (\ln x(90) - 1.2)/0.4) = 0.1, \) so \( \ln x(90) - 1.2)/0.4 = 1.28, \) and solving for \( x(90) \) we get: \( x(90) = 5.54 \). So, the 90\(^{th}\) percentile of the waiting times is 5.54 hrs.