MATH/STAT 352, FALL 2013, PRACTICE EXERCISES FOR TEST 2

INSTRUCTIONS: Please try and do these problems without looking at the book. Solutions are provided at the end of this document. Your test questions will differ from this practice questions. You will need the normal and t-distribution tables for the exam - make sure you have copies with you! All questions that appear below, and those on the test, may be asked as open-ended or multiple choice questions.

ENJOY!

1. T or F. A 99% confidence interval for the population mean $\mu$ is determined to be (65.32 to 73.54). If the confidence level is reduced to 90%, the 90% confidence interval for $\mu$ becomes narrower.

2. An electrical firm that manufactures a certain type of bulb wants to estimate its mean life. Assuming that the life of the bulbs has a normal distribution with a standard deviation $\sigma = 40$ hours, find how many bulbs should be tested so as to be 90 percent confident that the sample mean will not differ from the true mean life by more than 10 hours.

3. Suppose that the weight of golf balls is normally distributed with mean of 9.5 grams and standard deviation of 1.2 grams. I pack 36 golf balls into a bag. Find the probability that the bag will weigh less than 360 grams? (Hint: Use the Central Limit Theorem)

4. A CI is desired for the true average stray-load loss $\mu$ (watts) for a certain type of induction motor when the line current is held at 10 amps for a speed of 1500 rpm. Assume that stray-load loss is normally distributed. Compute a 95% CI for $\mu$ when $n = 25$, $s=3.0$, and $\bar{x} = 60$.

5. A random sample of 100 lightning flashes in a certain region resulted in a sample average radar echo duration of .81 sec and a sample standard deviation of .34 sec. Calculate a 99% confidence interval for the true average echo duration $\mu$, and interpret the resulting interval.

6. A random sample of $n = 8$ E-glass fiber test specimens of a certain type yielded a sample mean interfacial shear yield stress of 30.5 and a sample standard deviation of 3.0. Assuming that interfacial shear yield stress is normally distributed, compute a 95% CI for true average stress.

7. In their annual report, the State Department of Education stated that 62% of academic employees in the State were permanent faculty and the remaining 38% were temporary. A professor in Bumbleton State University (BSU) claims that BSU has a lower proportion of permanent faculty than the overall State proportion of 62%. A random sample of 400 academic employees at BSU showed 215 of them to have permanent positions.
   a. Find a 90% confidence interval for the true proportion of permanent academic employees at BSU.
   b. How large a sample is required in order to be 99% confident that the sample proportion will not differ from the true proportion of permanent faculty at BSU by more than 0.02. Use the sample information stated in the problem for your initial estimate of $p$.
   c. Assume that no prior information about the proportion of permanent faculty at BSU is available. How large a sample is required to be 99% confident that the sample proportion of permanent faculty at BSU will not differ from the true proportion by more than 0.02.
8. A breeder of rabbits claims that he can breed rabbits yielding a mean weight of greater than 58 ounces. A random sample of 16 rabbits had a mean weight of 59.2 ounces and standard deviation of weights $s = 3$ ounces. Assume normal distribution of rabbits’ weights. Find a 95% confidence interval for the true mean weight of a rabbit.

9. A brewery distributes beer in cans labeled 12 oz. The Bureau of Weights and Measures randomly selects 36 cans, measures their mean contents, and finds their mean weight to be 11.82 oz. Assume that $\sigma$ is known to be 0.38 oz.

   a. Find a 95% confidence interval for the true mean content of a can.

   b. How large a sample is required in order to be 95% confident that the sample mean will not differ from the true mean content by more than 0.01.

10. The time $X$ (minutes) for a lab assistant to prepare the equipment for a certain experiment is believed to have a lognormal distribution with parameters $\mu = 1.2$ hrs and $\sigma = 0.4$ hrs.

   a. What is the mean prep time?

   b. What is the probability that preparation time exceeds 3 hrs?

   c. What is the 90th percentile of the prep times?

11. Let $Z$ be a standard normal random variable and calculate the following probabilities:

   a. $P(0 \leq Z \leq 2.25)$

   b. $P(0 \leq Z \leq 1)$

   c. $P(-2.0 \leq Z \leq 0)$

   d. $P(-1.50 \leq Z \leq 1.50)$

   e. $P(Z \leq 1.32)$

   f. $P(Z \geq -1.70)$

   g. $P(-1.50 \leq Z \leq 2.00)$

   h. $P(1.50 \leq Z \leq 2.50)$

   i. $P(Z \geq 1.50)$

   j. $P(Z \leq 2.40)$

12. If $X$ is a normal random variable with mean 85 and standard deviation 10, compute the following probabilities.

   a. $P(X \leq 100)$

   b. $P(X \leq 80)$

   c. $P(65 \leq X \leq 100)$

   d. $P(X \geq 70)$

   e. $P(85 \leq X \leq 95)$

   f. $P(X - 80 \leq 10)$

13. Suppose only 40% of all drivers in Florida regularly wear a seatbelt. A random sample of 500 drivers is selected. What is the probability that

   a. Between 170 and 220 (inclusive) of the drivers in the sample regularly wear a seatbelt?

   b. Fewer than 175 of those in the sample regularly wear a seatbelt?

14. Let $X_1, X_2, \ldots, X_{100}$ denote the actual net weights in lb of 100 randomly selected 50-lb bags of fertilizer.

   a. If the expected weight of each bag is 50 and the variance is 1, calculate $P(49.8 \leq \bar{X} \leq 50.3)$ (approximately) using the CLT.
b. If the expected weight of each bag is 49.8 lb rather than 50 lb so that on average bags are underfilled, calculate \( P(49.8 \leq \bar{X} \leq 50.3) \).

15. A physical fitness association is including the mile run in its secondary-school fitness test for boys. The time for this event for boys in secondary school is normally distributed with mean 500 seconds and a standard deviation of 45 seconds.
   a. What percentage of the boys finish the mile run in less than 400 seconds?
   b. What percentage of the boys take between 400 and 590 seconds to finish the mile run?
   c. If the association wants to designate the fastest 1% as "excellent," what time (in sec) should the association set for this criterion?

16. The time \( X \) (minutes) for a lab assistant to prepare the equipment for a certain experiment is believed to have a uniform distribution with \( A = 20 \) and \( B = 30 \).
   a. Write the pdf of \( X \) and sketch its graph.
   b. What is the probability that preparation time exceeds 27 minutes?
   c. Find the preparation mean time, then calculate the probability that preparation is within 2 minutes of the mean time?

17. Let \( X \) = the time between two successive arrivals at the drive-up window of a local bank. If \( X \) has an exponential distribution with \( \lambda = 1 \), compute the following:
   a. The expected time between two successive arrivals.
   b. The standard deviation of the time between two successive arrivals.
   c. \( P(X \leq 5) \).
   d. \( P(3 \leq X \leq 5) \).

18. Let \( X \) = hourly median power (in decibels) of received radio signals transmitted between two cities. It is believed that the lognormal distribution provides a reasonable probability model for \( X \). If the parameter values are \( \mu = 3.5 \) and \( \sigma = 1.2 \), calculate the following:
   a. The mean value and standard deviation of received power
   b. The probability that received power is between 50 and 250 dB
   c. The probability that \( X \) is less than its mean value. Why is this probability not .5?
**Problem 1.** T.

**Problem 2.** Here \( \sigma=40, \ m=10, \ \alpha=0.1, \) so \( \alpha/2=0.05. \) Thus \( n = \left(\frac{Z_{\alpha/2}\sigma}{m}\right)^2 = \left(\frac{1.645 \ (40)}{10}\right)^2 = 43.3, \) round up \( n=44. \)

**Problem 3.** Weights of golf balls have a normal distribution with mean=9.5, and standard deviation =1.2 grams. Then, the sample mean of 36 balls \( \bar{X} \sim N \left(9.5, \frac{1.2^2}{36}\right) = N(9.5, 0.04). \) Thus
\[
P(\text{total weight} < 360) = P\left(\bar{X} < \frac{360}{36}\right) = P(\bar{X} < 10) = P(Z < (10 - 9.5)/ 0.2) = P(Z < 2.5)=0.9938
\]

**Problem 4.** \( \mu= \) mean stray-loss (watts), normal population of stray losses.

95% CI for \( \mu: \bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} = \) I used MINITAB = (58.762, 61.238). If you were using the formula and calculator, then use \( t(0.025, 24) = 2.064 \)

**IN MINITAB: One-Sample T**

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>60.00</td>
<td>3.00</td>
<td>0.60</td>
<td>(58.762, 61.238)</td>
</tr>
</tbody>
</table>

**Problem 5.** 99% CI for \( \mu= \) mean length of radar echo:

\( \bar{X} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}} = \) I used MINITAB = (0.7207, 0.8993). If I needed to use the t-table, I need \( t \) with 99 df which is not listed. I would have used \( t \) distribution with \( df=120 \) which is available and closest to 99 df. I would get \( t(0.005) = 2.617. \) The CI would be a little different than the one done in MINITAB. The reason is that MINITAB uses exact value of \( t(0.005, 0.005). \) We could also use \( z \)-percentile in place of \( t \)-percentile, because the sample size is large.

**IN MINITAB: One-Sample T**

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>99% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.8100</td>
<td>0.3400</td>
<td>0.0340</td>
<td>(0.7207, 0.8993)</td>
</tr>
</tbody>
</table>

**Problem 6.** 95% CI for \( \mu= \) mean stress:

\[
d.f. = n - 1 = 7, \text{ so the critical value for a } 95\% \text{ C.I. is } t_{0.025, 7} = 2.365 = 2.365. \text{ The interval is then}
\]
\[
30.5 \pm (2.365) \left(\frac{3.0}{\sqrt{8}}\right) = 30.5 \pm 2.51(27.99, 33.01).
\]

**Problem 7.**

a. Let \( p= \) the true proportion of permanent academic employees at BSU. We need a 90% confidence interval for \( p. \)

I used the traditional formula: \( \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \) with
\[ \hat{p} = \frac{215}{400} = 0.5375, \quad n = 400, \quad z_{a/2} = 1.645, \text{ so } 90\% \text{ CI for p is: } \left( 0.4965, 0.5785 \right) \]

**IN MINITAB:** result a bit different due to rounding

**Test and CI for One Proportion**

<table>
<thead>
<tr>
<th>Sample</th>
<th>X</th>
<th>N</th>
<th>Sample p</th>
<th>90% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>215</td>
<td>400</td>
<td>0.537500</td>
<td>(0.495138, 0.579447)</td>
</tr>
</tbody>
</table>

b. How large a sample is required in order to be 99% confident that the sample proportion will not differ from the true proportion of permanent faculty at BSU by more than 0.02. Use the sample information stated in the problem for your initial estimate of \( p \).

I used the traditional formula: 

\[ n = \frac{z_{a/2}^2 \hat{p}(1-\hat{p})}{m^2} \]

with \( \hat{p} = 0.5375, \quad z_{a/2} = 2.576, \quad \text{and } m = 0.02, \)

we get \( n = 4125 \)

c. Assume that no prior information about the proportion of permanent faculty at BSU is available. How large a sample is required to be 99% confident that the sample proportion of permanent faculty at BSU will not differ from the true proportion by more than 0.02.

Since no prior information for \( p_0 \), we use \( \hat{p} = \frac{1}{2} \):

\[ n = \frac{2.576^2(0.5)(1-0.5)}{0.02^2} = 4147.36 \approx 4148 \]

**Problem 8.** Rabbit problem;

The 95% confidence interval for \( \mu \) is

\[
\left[ \bar{x} - t(\nu, 0.05)\left(\frac{s}{\sqrt{n}}\right), \quad \bar{x} + t(\nu, 0.05)\left(\frac{s}{\sqrt{n}}\right) \right]
\]

\[= \left[ 59.2 - t(15, 0.025)\left(\frac{3}{4}\right), \quad 59.2 + t(15, 0.025)\left(\frac{3}{4}\right) \right]
\]

\[= \left[ 59.2 - 2.131*0.75, \quad 59.2 + 2.131*0.75 \right]
\]

\[= \left[ 57.60, \quad 60.79 \right]
\]

**Problem 9.** Brewery problem.

a. let \( \mu = \text{true mean content of a can. The 95\% CI for } \mu \) is:

\[
(\bar{X} \pm Z\alpha/2 \cdot \frac{\sigma}{\sqrt{n}}) = (11.82 \pm (1.96)(0.38/\sqrt{36})) = (11.696, 11.944)
\]

b. The sample size is:

\[ n = [z(\alpha/2)\cdot(\sigma/m)]^2 = [(-1.96)(0.38)/(0.01)]^2 = 5547.2704 \]

Hence, we would need a sample size of \( n = 5548 \) to be 95% confident that the sample mean will not differ from the true mean content by more than 0.01 (margin of error).

**Problem 10.** Let \( X = \text{prep time in hours, } X \sim \text{lognormal}(1.2, 0.4^2) \). Let \( Y = \ln X \sim N(1.2, 0.4^2) \)

a. \( EX = e^{\mu+\sigma^2/2} = e^{1.2+0.4^2/2} = 3.597 \). The mean prep time is 3.6 hrs or 3 hrs and 36 min.

b. \( P(X > 3) = P(\ln X > \ln 3) = P(Y > 1.1) = P(Z > -0.25) = 0.5987 \).
c. Let $x(90)=90^{th}$ percentile of the prep times. That is $P(X > x(90)) = 0.1$. Thus, $P(\ln X > \ln(x(90)))=0.1$, standardizing we get:

$$P(Z > (\ln(x(90))-1.2)/0.4 )=0.1,$$

so $\ln(x(90))-1.2)/0.4= 1.28$, and solving for $x(90)$ we get: $x(90)=5.54$.

So, the $90^{th}$ percentile of the prep times is 5.54 hrs.

**Problem 11.** Let $\Phi(x)=$ standard normal cdf at $x$, that is $\Phi(x)=P(Z \leq x)$.

a. $P(0 \leq Z \leq 2.25) = \Phi(2.25) - \Phi(0) = .4878$

b. $P(0 \leq Z \leq 1) = \Phi(1) - \Phi(0) = .3413$

c. $P(-2.0 \leq Z \leq 0) = \Phi(0) - \Phi(-2.0) = .4772$

d. $P(-1.50 \leq Z \leq 1.50) = \Phi(1.50) - \Phi(-1.50) = .8664$

e. $P(Z \leq 1.32) = \Phi(1.32) = .9066$

f. $P(Z \geq -1.70) = 1 - \Phi(-1.70) = .9554$

g. $P(-1.50 \leq Z \leq 2.00) = \Phi(2) - \Phi(-1.50) = .9104$

h. $P(1.50 \leq Z \leq 2.50) = \Phi(2.50) - \Phi(1.50) = .0606$

i. $P(Z \geq 1.50) = 1 - \Phi(1.50) = .0668$

j. $P(Z \leq 2.40) = P(-2.40 \leq Z \leq 2.40) = \Phi(2.40) - \Phi(-2.40) = .9836$

**Problem 12.** \(X \sim N(85, 10)\)

a. \(P(X \leq 100) = P(Z \leq 1.5) = \Phi(1.50) = .9332\)

b. \(P(X \leq 80) = P(Z \leq -.5) = \Phi(-.5) = .3085\)

c. \(P(65 \leq X \leq 100) = P(-2 \leq Z \leq 1.5) = \Phi(1.5) - \Phi(-2.0) = .9332 - .0228 = .9104\)

d. \(P(X \geq 70) = P(Z \geq -1.50) = 1 - \Phi(-1.50) = .9332\)

e. \(P(85 \leq X \leq 95) = P(0 \leq Z \leq 1.0) = \Phi(1.0) - \Phi(0) = .3413\)

f. \(P(X = 80 \leq 10) = P(-10 \leq X - 80 \leq 10) = P(70 \leq X \leq 90) = P(-1.5 \leq Z \leq .5) = \Phi(.5) - \Phi(-1.5) = .6915 - .0668 = .6247\)

**Problem 13.** \(n = 500, \rho = .4, \mu = 200, \sigma = 10.9545\). Use normal approximation to binomial. Use \(Y \sim N(200, 10.9545^2)\) as an approximation to \(X \sim \text{Bin}(500, 0.4)\). Remember about the continuity correction.

a. \(P(170 \leq X \leq 220) \approx P(169.5 < X < 220.5) \approx P(-2.87 < Z < 1.87) = 0.966.\)

b. \(P(X < 175) = P(X \leq 174) \approx P(Y < 174.5) = P(Z < -2.33) = 0.0099\)

**Problem 14.** Using CLT we approximate the distribution of \(\bar{X}\) by \(N(50, 0.12)\).

a. \(\mu_x = \mu = 50, \sigma_x = \frac{\sigma}{\sqrt{n}} = 10\),

\[P(49.8 \leq \bar{X} \leq 50.3) = P\left( \frac{49.8 - 50}{10} \leq Z \leq \frac{50.3 - 50}{10} \right) = P(-2.0 \leq Z \leq 3.0) = .9759\]

b. \(P(49.8 \leq \bar{X} \leq 50.3) \approx P\left( \frac{49.8 - 49.8}{10} \leq Z \leq \frac{50.3 - 49.8}{10} \right) = P(0 \leq Z \leq 5) = .5000\)
**Problem 15.** Let $T=$ time a boy in secondary school takes to do a mile run. $T$ has $N(500, 45)$ distribution.

a. Percent that finish in less than 400 sec.

$$P(T< 400) = P((T-500)/45 < (400-500)/45)=P(Z< -2.22)=0.0132$$

b. Percent that finish b/w 400 and 590 sec. $P(400 < T< 590) = P(-2.22< Z < 2)= 0.9772 -0.0132 = 0.964$

c. Find 99th percentile of the run times. 99th percentile of the standard normal distribution is 2.33.

$$2.33=(T-500)/45$$ solving for $T$, we get $T= 604.85$, so the 99th percentile of the run times is 604.85 sec.

**Problem 16.** The time $X$ (minutes) for a lab assistant to prepare the equipment; $X \sim U(20, 30)$.

a) The pdf of $X$ is $f(x) = 0.1$ if $20 < X < 30$, and 0 otherwise.

b) $P(X > 27) = (0.1) * 3 = 0.3$

c) $EX = (20 + 30)/2= 25$ min

d) $P(23 < X < 27) = (27 – 23)* 0.1=0.4$

**Problem 17.** a. $E(X) = 1/\lambda =1$

b. $\sigma = 1/\lambda =1$

c. $P(X \leq 5) = 1 – e^{-(1)(5)} = 1 – e^{-5} = .9933$

d. $P(3 \leq X \leq 5) = 1 – e^{-(1)(5)} – \left[ 1 – e^{-(1)(3)} \right] = e^{-3} – e^{-5} = .043$

**Problem 18.**

a. $E(X) = e^{3.5+ (1.2) / 2} = 68.0335; V(X) = e^{2(3.5) + (1.2) \cdot (3.5)} - 1 = 14907.168; \text{ and } \sigma_x = 122.0949$

b. Let $Y \sim N(3.5, 1.2)$, then $P(50 < X < 250) = P(\ln50 < Y < \ln250)=P(0.3 < Z < 1.68) = 0.3204$

c. $P(X < 68.0335) = (P(Z < 0.6) = 0.7257$. This probability is not 0.5 because the lognormal distribution is not symmetric.