Instructions:

- Each part of every problem is worth 4 points. You receive 4 points for a perfectly done problem; partial credit will be given for work that is not perfectly correct, but contains elements that are correct and relevant to the problem and does not contain major mistakes; 0 points otherwise.

- Please show your calculations and justify your claims, write on one side of the paper, and turn problems in in the order they appear on this paper.

- I will use these exercises as a tool to make decisions in "border" cases.

1. The density function of the random variable $X$ is given by.

$$f_X(x) = \begin{cases} 
\frac{1}{2} & \text{for } -1 < x < 1, \\
0 & \text{otherwise}.
\end{cases}$$

(a) Determine the moment generating function of $X$.

(b) Determine $E(X)$.

(c) Let $X_1, X_2, \ldots, X_{20}$ be iid with the same distribution as $X$. Find the moment generating function of $Y = \sum_{i=1}^{20} X_i$.

2. Suppose that the proportion of colorblind people in a certain population is 0.005. What is the probability that there will be no more that 1 colorblind person in a randomly chosen group of 600 people?

3. Suppose that a sequence of independent tosses are made with a coin for which the probability of obtaining a head on a given toss is $\frac{1}{30}$.

   (a) What is the expected number of tails that will be obtained before five heads have been obtained?

   (b) What is the variance of the number of tails that will be obtained before five heads have been obtained?

4. Suppose that $X$ has a geometric distribution with parameter $p$. Determine the probability that the value of $X$ is an even integer 0, 2, 4, $\ldots$, that is $P(X = 0$ or $X = 2$ or $X = 4$ or $\ldots$).