Problem 6.2.3.

(a) Suppose \( H_o : \mu = \mu_o \) rejected in favor of \( H_1 : \mu \neq \mu_o \) on \( \alpha = 0.05 \) significance level. Would \( H_o \) be necessarily rejected at \( \alpha = 0.01 \) level of significance?

No. Since \( H_o \) rejected at the \( \alpha = 0.05 \) significance level, then p-value < 0.05. However, p-value could be for example 0.03, and thus \( H_o \) would not be rejected at the \( \alpha = 0.01 \) significance level.

(b) Suppose \( H_o : \mu = \mu_o \) rejected in favor of \( H_1 : \mu \neq \mu_o \) on \( \alpha = 0.01 \) significance level. Would \( H_o \) be necessarily rejected at \( \alpha = 0.05 \) level of significance?

Yes. Since \( H_o \) rejected at the \( \alpha = 0.01 \) significance level, then p-value < 0.01. Thus p-value < 0.05.

Problem 6.2.6.

Recall that the significance level \( \alpha \) is the probability to reject \( H_o \), i.e. for the sample mean to fall in the critical region \( C \in (29.9, 30.1) \) given that \( H_0 \) is true. That is

\[
\alpha = P \left( \frac{29.9 - 30}{6/\sqrt{16}} \leq Z \leq \frac{30.1 - 30}{6/\sqrt{16}} \right) = P(-0.0667 \leq Z \leq 0.0667) = 0.5265897 - 0.4734103 \approx 0.0532
\]

\( C = (29.9, 30.1) \) is a poor choice for the critical region because it rejects \( H_0 \) for \( \bar{y} \) values that are close to the true mean if \( H_0 \) is true. That is, it rejects \( H_0 \) for \( \bar{y} \) values that are most compatible with \( H_0 \).

Note that \( Z_{\alpha/2} = Z_{0.0532/2} = Z_{0.0266} = \pm 1.935 \), so since we have a two sided test, \( H_0 \) should be rejected if

\[
\bar{y} < 30 - 1.935\frac{6}{\sqrt{16}} \text{ OR } \bar{y} > 30 + 1.935\frac{6}{\sqrt{16}} . \text{ Final decision rule is to reject } H_0 \text{ if}
\]

\[
\bar{y} \in (-\infty, 27.0975] \cup [32.9025, \infty)
\]

Problem 6.2.7.

(a) We have 30 measurements. Let \( \mu \) = true mean alcohol level reading for a person with blood alcohol level of 12.6%. We need to test:

\( H_o : \mu = 12.6 \) versus \( H_1 : \mu \neq 12.6 \)

on \( \alpha = 0.05 \) significance level. Assume the standard deviation of the readings is \( \sigma = 0.4 \). Would you recommend that the testing machine be readjusted?

From the sample we get \( \bar{x} = 12.76 \), so the test statistic

\[
z = \frac{12.76 - 12.6}{0.4/\sqrt{30}} = 2.19.
\]
The CR for the test is $|z| > 1.96$. Our test statistic falls into the CR, so we reject $H_0$. Machine should be readjusted.

(b) Assume $X_1, X_2, \ldots, X_{30}$ are iid from $N(\mu, 0.4)$. To check the normality assumption we should look at a histogram of probability plot for the data. We can not test for independence, but since the subjects were different, we can assume that the measurements were independent.

**Problem 6.2.10.**

Let $\mu$ be the true mean blood pressure when taking exams. Since Rosaura is interested in examining whether the stress of final exams elevates the blood pressure of freshmen women, the appropriate hypothesis would be:

$$
H_0 : \mu = 120 \quad H_1 : \mu > 120
$$

Note that $n = 50, \sigma = 12, \mu = 120, \text{ and } \bar{x} = 125.2$. So, the test statistic $Z$ is

$$
Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{125.2 - 120}{12/\sqrt{50}} \approx 3.064
$$

Now, since $p\text{-value}=P(Z \geq 3.064)) = 0.001092$, then $H_0$ would be rejected for any $\alpha > 0.001092.$