STAT 452/652 HOMEWORK 2 SOLUTIONS

1. What happens to the histograms as we increase the sample size?

As the sample size increases, the histogram becomes smoother. It approaches/resembles the density curve of the distribution the data came from, as evidenced by the close fit of the density curves of the normal distribution to the histograms on the above plots.

2. X has exponential distribution with mean 2. I used MINITAB for all the calculations.
   a. $P(X > 3) = 1 - 0.7769 = 0.2231$
   b. 20th percentile of X is 0.4463.

3. X has Binomial distribution with n=20 and p=0.7.
   a. $P(5 < X \leq 12) = P(X \leq 12) - P(X \leq 5) = 0.22773 - 0.000043 = 0.227687$
   b. 65th percentile of X is 15.
   c. 50th percentile of X is 14.

4. **452 students only.** Observation from U(0,1) is 0.977. By FTSS, to get an observation from chi-square distribution with 10 df, I need to find $F^{-1}(0.977)$ (inverse cdf), where F is the cdf of chi-square rv with 10 df. I will do that using MINITAB, to get: 20.7375. Answer: the observation from chi-square distribution with 10 df is 20.7375.

5. **652 students only.** Observation from U(0,1) is 0.5. By FTSS, to get an observation from Pareto distribution with $\alpha=2$, I need to find $F^{-1}(0.5)$ (inverse cdf), where F is the cdf of a Pareto rv with $\alpha=2$. First, I find F and then $F^{-1}$. Using integration, the cdf of a Pareto rv with $\alpha=2$ is

   \[ F(x) = \int_1^x \frac{2}{t^3} dt = 1 - \frac{1}{x^2} \] for $x \geq 1$. Next, we compute

   \[ F^{-1}(x) = \sqrt{\frac{1}{1-x}} \] for $0 < x < 1$. Finally, the required observation from Pareto is $F^{-1}(0.5) = \sqrt{2}$. 