1. Suppose that only 25% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when no other cars are visible. What is the probability that, of 20 randomly chosen drivers coming to an intersection under these conditions,

a. At most 6 will come to a complete stop?
b. Exactly 6 will come to a complete stop?
c. How many of the next 20 drivers do you expect to come to a complete stop?

**ANSWER:** Let \( S = \# \) of drivers that come to a complete stop out of 20, so \( S \sim \text{Bin}(20, 0.25) \) \( p = .25 \), \( n = 20 \). Let \( B(x, 20, 0.25) \) denote cdf of \( X \) at \( x \), that is \( P(X \leq x) = B(x, 20, 0.25) \). Then (using tables)

a. \( P(X \leq 6) = B(6; 20, .25) = .786 \)
b. \( P(X = 6) = b(6; 20,.25) = B(6; 20,.25) - B(5; 20,.25) = .786 -.617 = .169 \)
d. \( E(X) = (20)(.25) = 5. \) We expect 5 of the next 20 to stop.

2. The results of an entry test for a data processing company have a normal distribution with mean 60 and standard deviation 10. Only the top 2 percent of applicants who took the test will be hired.

a. What is the lowest score of a hired applicant?

**ANSWER:** Let \( X \) be the score on a test for an applicant. Then \( X \sim \text{N}(60, 10) \). Let \( a \) be the lowest score of a hired applicant, that is \( a \) is the 98th percentile of the distribution of \( X \). Then

\[
0.98 = P(X \leq a) = P\left(Z \leq \frac{a - 60}{10}\right) \Rightarrow \frac{a - 60}{10} = 2.05 \quad \text{and} \quad a = (2.05)(10) + 60 = 80.5. \quad \text{So, the lowest score of a hired applicant is 80.5.}
\]

b. What percentage of applicants scored between 50 and 65?

**ANSWER:** \( P(50 < X < 65) = P\left(\frac{50 - 60}{10} < Z < \frac{65 - 60}{10}\right) = P(-1 < Z < 0.5) = 0.533. \)

4. Let \( X \) = hourly median power (in decibels) of received radio signals transmitted between two cities. It is believed that the lognormal distribution provides a reasonable probability model for \( X \). If the parameter values are \( \mu = 3.5 \) and \( \sigma = 1.2 \), calculate the following:

a. The mean value of received power
b. The probability that received power is between 50 and 250 dB

**ANSWER:**

a. \( E(X) = e^{3.5+1.2^2/2} = 68.0335; \)

b. \( P(50 \leq X \leq 250) = P\left(\frac{\ln(50) - 3.5}{1.2} \leq Z \leq \frac{\ln(250) - 3.5}{1.2}\right) = P(Z \leq 1.68) - P(Z \leq .34) = .9535 - .6331 = .3204. \)

5. Extensive experience with fans of a certain type used in diesel engines has suggested that the exponential distribution provides a good model for time until failure. Suppose the mean time until failure is 25,000 hours. What is the probability that

a. A randomly selected fan will last between 20,000 and 30,000 hours?
b. The lifetime of a fan exceeds the mean value by more than 2 standard deviations?

**ANSWER:** Mean = \( \frac{1}{\lambda} = 25,000 \) implies \( \lambda = .00004 \)

a. \( P(X > 20,000) = 1 - P(X \leq 20,000) = 1 - F(20,000; .00004) = e^{-1000000(20,000)} = .449 \)

b. \( P(X \leq 30,000) = F(30,000; .00004) = e^{-1.2} = .699 \), \( P(20,000 \leq X \leq 30,000) = .699 - .551 = .148 \)
6. A farmer divided his field into a large number of small plots of the same size. The amount of yield per plot has a mean of 100 bushels with a standard deviation of 8 bushels. If 64 plots are selected at random, find the probability that the mean yield will be between 98 and 102 bushels.

**ANSWER:** Let $X$ be the yield per plot. $EX = 100$, $\sigma(X) = 8$. 64 plots selected, yields from 64 plots observed: $X_1, \ldots, X_{64}$. $\bar{X} = \frac{1}{64}(X_1 + \ldots + X_{64})$ is the mean yield per plot. To find probability connected with $\bar{X}$ we need its distribution. To get distribution of the mean, we need to use the Central Limit Theorem. We will get an approximate distribution of $\bar{X}$. $\bar{X} \sim N(100, \frac{8}{\sqrt{64}})$ i.e. $\bar{X} \sim N(100, 1)$ approximately. Thus, 

$$P(98 < \bar{X} < 102) \approx P\left(\frac{98 - 100}{1} < Z < \frac{102 - 100}{1}\right) = P(-2 < Z < 2) = 0.9772 - 0.0228 = 0.9544$$