Physics 152  
Exam 4  
Summer 1997

1. A "mission controller" and a pilot are both planning a space trip to and from a distant star. The controller has had training only in classical physics and doesn't know about time dilation. He recommends taking supplies for 100 years. The pilot understands relativity perfectly and makes plans for only 20 years worth of supplies.

a) at what speed will the spacecraft be traveling?

$$t = \frac{100}{13} \text{ yrs}$$
$$t_0 = 20 \text{ yrs}$$

$$100 = \frac{20}{13}$$

$$\beta = \frac{2}{\sqrt{2}}$$

$$\frac{v^2}{c^2} = 1 - \frac{\beta^2}{c^2}$$

$$v = \sqrt{0.96} c$$

b) how far is the star from earth (one way)?

**Lo is earth measurement (in reality) (longer length - proper length)**

$$L_0 = v t = (0.98)(c)(100 \text{ yrs}) = 98 \text{ light yrs}.$$  

B/c one way $$D = \frac{L_0}{2} = 49 \text{ light yrs}.$$  

c) how far will the spacecraft travel in its frame of reference?

**L is frame of reference (shorter length)**

$$L = L_0 \beta = 0.2 (49 \text{ light yrs})$$

$$= 9.8 \text{ light years}$$

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\* V is the same for both of them!
\* L = v t  will also work
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d) if the spacecraft has a rest mass of 5000kg., how much energy must be expelled to get it "up to speed"?

$$m_0 = 5000$$

$$KE = m_0 c^2 \left( \frac{1}{\beta} - 1 \right)$$

$$= 5000 \left( 9 \times 10^{16} \right) \left( \frac{1}{0.2} - 1 \right)$$

$$= 4 \left( 5000 \right) \left( 9 \times 10^{16} \right)$$

$$= 1.8 \times 10^{21}$$
2. An electron is accelerated from rest to a speed of .995c. 
   a) What is it's mass at this speed?

   \[ m = \frac{m_0}{\beta} \]
   \[ \beta = \sqrt{1 - (0.995)^2} = 0.0999 \]
   \[ = 9.1 \times 10^{-31} \text{ kg} \]

   b) Through what potential was it accelerated to achieve this speed?

   \[ KE = m_0 c^2 \left( \frac{1}{\beta} - 1 \right) \]  
   \[ = (0.11 \text{ MeV}) \left( \frac{1}{0.999} - 1 \right) \]
   \[ = 9.599 \times 10^6 \text{ MeV} \]

3. Light of wavelength 540 nm strikes a metallic surface and electrons are emitted that require a stopping potential of .90 volts.
   a) What is the work function of the metal?

   \[ \frac{\hbar c}{\lambda} - \phi = KE_{\text{max}} = eV_{\text{stopping}} \]
   \[ \left( 4.14 \times 10^{-15} \text{ (eV} \cdot \text{sec}) \right) \left( 3 \times 10^8 \text{ m/s} \right) \]
   \[ \frac{540 \times 10^{-9}}{950 \times 10^{-9}} \]
   \[ - \phi = 0.90 \text{ eV} \]

   b) What is the speed of the fastest electrons emitted when a light source of 450 nm is used instead?

   \[ \frac{\hbar c}{\lambda} - \phi = \frac{1}{2} mv_{\text{max}}^2 \]
   \[ \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{950 \times 10^{-9}} - 1.4 \times 10^{-19} = 6 \times 10^{-34} \]
   \[ 4.52 \times 10^{-19} - 2.24 \times 10^{-19} = 4.28 \times 10^{-19} \]
   \[ v_{\text{max}}^2 = 4.28 \times 10^{-19} \]
   \[ v_{\text{max}} = 6.9 \times 10^6 \]

4. An electron microscope that has the same resolution as X-rays of 8 = .01 nm is desired. Through what potential must these electrons be accelerated to have the same wavelength?

   \[ \lambda_{DB} = 0.01 \times 10^{-9} \text{ m} = \frac{h}{mv} \]
   \[ V = \frac{h}{m} \left( \frac{0.1 \times 10^{-9}}{c \cdot \beta - 1} \right) \]
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   \[ \frac{v^2}{h^2} \left( 1 - \frac{v^2}{c^2} \right)^{1/2} = \frac{\left( 9.1 \times 10^{-31} \text{ (eV} \cdot \text{sec}) \right) \left( 0.01 \times 10^{-9} \right)}{\left( 4.14 \times 10^{-15} \text{ (eV} \cdot \text{sec}) \right) \left( 3 \times 10^8 \text{ m/s} \right)} \]
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   \[ \frac{v^2}{h^2} \left( 1 - \frac{v^2}{c^2} \right)^{1/2} = \frac{\left( 9.1 \times 10^{-31} \text{ (eV} \cdot \text{sec}) \right) \left( 0.01 \times 10^{-9} \right)}{\left( 4.14 \times 10^{-15} \text{ (eV} \cdot \text{sec}) \right) \left( 3 \times 10^8 \text{ m/s} \right)} \]
   \[ \lambda_{DB} = 0.01 \times 10^{-9} \text{ m} = \frac{h}{mv} \]
5. Light consisting of wavelengths 693nm. and 459nm. passes through a diffraction grating with 5700 lines per cm. Find the distance between the 1st order 459 line and the 2nd order 693 line on a screen 2.5 meters from the grating.

\[
\sin \theta = \frac{m \lambda}{d} \quad \tan \theta = \frac{bm}{0}
\]

\[
\sin \theta_{1,693} = \left(\frac{693 \times 10^{-9}}{570000}\right) = 1.0 \times 10^{-9} = \theta_{1,693} = 15.2^\circ
\]

\[
d = \frac{1}{5700} \times 10^{-6} \quad \text{or} \quad \frac{1}{d} = 570000
\]

6. A man who has a near point of 20 cm without glasses but a near point of 45 cm when he wears his "distance correcting" glasses. What was his far point without glasses?

\[d_0 = 45 \text{ cm} \quad d_1 = -20 \text{ cm}\]

6. What is the longest wavelength (lowest energy) ultraviolet photon that can be emitted by hydrogen atoms?

\[n_P = 20 \text{ cm (without glasses)} \quad 50^\circ\]

\[\frac{1}{d_0} + \frac{1}{d_1} = \frac{1}{P} \quad \frac{1}{45} + \frac{1}{-20} = \frac{1}{P} \]

\[0.02 + 0.05 = \frac{1}{P} \quad -0.02 = \frac{1}{P} \quad F = -35.7 \text{ cm}
\]

*Make up bifocal prescription!*  
L -> for test!