3.4 Human eye  Photons passing through the pupil are focused by the lens onto the retina of the eye and are detected by two types of photosensitive cells, called rods and cones, as visualized in Figure 3.46. Rods are highly sensitive photoreceptors with a peak response at a wavelength of about 507 nm (green-cyan). They do not register color and are responsible for our vision under dimmed light conditions, termed scotopic vision. Cones are responsible for our color perception and daytime vision, called photopic vision. These three types of cone photoreceptors are sensitive to blue, green, and red at wavelengths, respectively, of 430 nm, 535 nm, and 575 nm. All three cones have an overall peak response at 555 nm (green), which represents the peak response of an average daylight-adapted eye or in our photopic vision.

(a) Calculate the photon energy (in eV) for the peak responsivity for each of the photoreceptors in the eye (one rod and three cones).

(b) Various experiments (the most well known being by Hecht et al., J. Opt. Soc. America, 38, 196, 1942) have tested the threshold sensitivity of the dark-adapted eye and have estimated that visual perception requires a minimum of roughly 90 photons to be incident onto the cornea in front of the eye’s pupil and within 1/10 second. Taking 90 incident photons every 100 ms as the threshold sensitivity, calculate the total photon flux (photons per second), total energy in eV (within 100 ms), and the optical power that is needed for threshold visual perception.

(c) Not all photons incident on the eye make it to the actual photoreceptors in the retina. It has been estimated that only 1 in 10 photons arriving at the eye’s cornea actually make it to rod photoreceptors, due to various reflections and absorptions in the eye and other loss mechanisms. Thus, only nine photons make it to photoreceptors on the retina. It is estimated that the nine test photons fall randomly onto a circular area of about 0.0025 mm². What is the estimated threshold intensity for visual perception? If there are 150,000 rods mm⁻² in this area of the eye, estimate the number of rods in this test spot. If there are a large number of rods, more than 100 in this spot, then it is likely that no single rod receives more than one photon since the nine photons arrive randomly. Thus, a rod must be able to sense a single photon, but it takes nine excited rods, somehow summed up by the visual system, to generate the visual sensation. Do you agree with the latter conclusion?

(d) It is estimated that at least 200,000 photons per second must be incident on the eye to generate a color sensation by exciting the cones. Assuming that this occurs at the peak sensitivity at 555 nm, and that as in part (b) only about 10 percent of the photons make it to the retina, estimate the threshold optical power stimulating the cones in the retina.

*3.6 X-rays, exposure, and roentgens  X-rays are widely used in many applications such as medical imaging, security scans, X-ray diffraction studies of crystals, and for examining defects such as cracks in objects and structures. X-rays are highly energetic photons that can easily penetrate and pass through various objects. Different materials attenuate X-rays differently, so when X-rays are passed through an object, the emerging X-rays can be recorded on a photographic film, or be captured by a modern flat panel X-ray image detector, to generate an X-ray image of the interior of the object; this is called radiography. X-rays also cause ionization in a medium and hence are known as ionization radiation. The amount of exposure (denoted by X) to X-rays, ionizing radiation, is measured in terms of the ionizing effects of the X-ray photons. One roentgen (1 R) is defined as an X-ray exposure that ionizes 1 cm³ of air to generate 0.33 nC of charge in this volume at standard temperature and pressure (STP). When a body is exposed to X-rays, it will receive a certain amount of radiation energy per unit area, called energy fluence $\Psi_E$, that is, so many joules per cm², that depends on the exposure $X$. If $X$ in roentgens is the exposure, then the energy fluence is given by
\[ \Psi_E = \left[ \frac{8.73 \times 10^{-6}}{\mu_{\text{en,air}}/\rho_{\text{air}}} \right] X \text{ J cm}^{-2} \]  \[ \text{[3.58]} \]

where \( \Psi_E \) is in J cm\(^{-2}\), and \( \mu_{\text{en,air}}/\rho_{\text{air}} \) is the mass energy absorption coefficient of air in cm\(^2\) g\(^{-1}\) at the photon energy \( E_{\text{ph}} \) of interest; the \( \mu_{\text{en,air}}/\rho_{\text{air}} \) values are listed in radiological tables. For example, for 1 R of exposure, \( X = 1 \), \( E_{\text{ph}} = 20 \text{ keV} \), and \( \mu_{\text{en,air}}/\rho_{\text{air}} = 0.539 \text{ cm}^2 \text{ g}^{-1} \).

Equation 3.58 gives \( \Psi_E = 1.62 \times 10^{-5} \text{ J cm}^{-2} \) incident on the object.

a. In mammography (X-ray imaging of the breasts for breast cancer), the average photon energy is about 20 keV, and the X-ray mean exposure is 12 mR. At \( E_{\text{ph}} = 20 \text{ keV} \), \( \mu_{\text{en,air}}/\rho_{\text{air}} = 0.539 \text{ cm}^2 \text{ g}^{-1} \). Find the mean energy incident per unit area in \( \mu\text{J cm}^{-2} \), and the mean number of X-ray photons incident per unit area (photons cm\(^{-2}\)), called photon fluence \( \Phi \).

b. In chest radiography, the average photon energy is about 60 keV, and the X-ray mean exposure is 300 \( \mu\text{R} \). At \( E_{\text{ph}} = 60 \text{ keV} \), \( \mu_{\text{en,air}}/\rho_{\text{air}} = 0.0304 \text{ cm}^2 \text{ g}^{-1} \). Find the mean energy incident per unit area in \( \mu\text{J cm}^{-2} \), and the mean number of X-ray photons incident per unit area.

c. A modern flat panel X-ray image detector is a large area image sensor that has numerous arrays of tiny pixels (millions) all tiled together to make one large continuous image sensor. Each pixel is an independent X-ray detector and converts the X-rays it receives to an electrical signal. Each tiny detector is responsible for capturing a small pixel of the whole image. (Typically, the image resolution is determined by the detector pixel size.) Each pixel in a particular experimental chest radiology X-ray sensor is 150 \( \mu\text{m} \times 150 \mu\text{m} \). If the mean exposure is 300 \( \mu\text{R} \), what is the number of photons received by each pixel detector? If each pixel is required to have at least 10 photons for an acceptable signal-to-noise ratio, what is the minimum exposure required in \( \mu\text{R} \)?

3.7 Photoelectric effect A photoelectric experiment indicates that violet light of wavelength 420 nm is the longest wavelength radiation that can cause photoemission of electrons from a particular multialkali photocathode surface.

a. What is the work function of the photocathode surface, in eV?

b. If a UV radiation of wavelength 300 nm is incident upon the photocathode surface, what will be the maximum kinetic energy of the photoemitted electrons, in eV?

c. Given that the UV light of wavelength 300 nm has an intensity of 20 mW/cm\(^2\), if the emitted electrons are collected by applying a positive bias to the opposite electrode, what will be the photoelectric current density in mA cm\(^{-2}\)?

3.12 Diffraction by X-rays and an electron beam Diffraction studies on a polycrystalline Al sample using X-rays gives the smallest diffraction angle \( 2\theta \) of 29.5° corresponding to diffraction from the (111) planes. The lattice parameter \( a \) of Al (FCC), is 0.405 nm. If we wish to obtain the same diffraction pattern (same angle) using an electron beam, what should be the voltage needed to accelerate the electron beam? Note that the interplanar separation \( d \) for planes \( (h,k,l) \) and the lattice parameter \( a \) for cubic crystals are related by

\[ d = a \left( \frac{h^2+k^2+l^2}{2} \right)^{1/2} \]

3.19 Atomic and ionic radii The maximum in the radial probability distribution of an electron in a hydrogen-like atom is given by Equation 3.44, that is, \( r_{\text{max}} = (n^2a_o)/Z \), for \( l = n - 1 \). The average distance \( \bar{r} \) of an electron from the nucleus can be calculated by using the definition of an average and the probability distribution function \( P_n,l(r) \), that is,

\[ \bar{r} = \int_0^\infty r P_n,l(r)dr = \frac{a_o n^2}{Z} \left[ \frac{3}{2} - \frac{l(l+1)}{2n^2} \right] \]

in which the right-hand side represents the result of the integration (which has been done by physicists). Calculate \( r_{\text{max}} \) and \( \bar{r} \) for the 2\( p \) valence electron in the B atom. Which value is closer to the radius of the B atom, 0.085 nm, given in the Period Table? Consider only the outermost electrons, and calculate \( r_{\text{average}} \) for Li, Li\(^+\), Be\(^{2+}\), and B, and compare with the experimental values of the atomic or ionic sizes: 0.15 nm for Li, 0.070 nm for Li\(^+\), 0.035 nm for Be\(^{2+}\), and 0.085 nm for B.