Problem Set 1

2.19

The volume of the air bubble at the bottom of the lake is

\[ V_1 = \left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3}\pi (1.5 \text{ cm})^3\right) (1 \text{ L} / 1000 \text{ cm}^3) = 1.41 \times 10^{-2} \text{ L} \]

At the bottom of the lake, \( P_1 V_1 = nRT_1 \)

At the surface of the lake, \( P_2 V_2 = nRT_2 \)

Since the number of moles, \( n \), in the bubble is the same at the surface and bottom,

\[ \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \]

The volume of the bubble at the surface is then

\[ V_2 = \frac{(P_1 V_1/T_1)(T_2/P_2)}{(2.8 \text{ atm})(1.41 \times 10^{-2} \text{ L})(273.15 + 25.0)K} = 4.18 \times 10^{-2} \text{ L} \]

We get the radius of the bubble at the surface from

\[ r^3 = \frac{9.98 \text{ cm}^3}{(4.18 \times 10^{-2} \text{ L}) (1000 \text{ cm}^3/1 \text{ L})} \] so \( r = 2.2 \text{ cm} \)

2.37

(a) The pressure experienced by the diver at a depth of 36 ft beneath the surface is

\[ P_1 = \text{atmospheric pressure} + \text{pressure exerted by 36 ft of seawater} \]

\[ = 1.0 \text{ atm} + (36 \text{ ft})(1 \text{ atm}/33 \text{ ft}) = 2.09 \text{ atm} \]

At constant \( n \) and \( T \), \( P_1 V_1 = P_2 V_2 \), so that the ratio between the volume of the diver’s lungs at 36 ft under the surface and that at the surface is

\[ V_2/V_1 = P_1/P_2 = (2.09 \text{ atm})/(1.0 \text{ atm}) = 2.1 \]

The diver’s lungs would expand by a factor of 2.1 when reaching the surface!

(b) At constant \( n \) and \( T \), \( V \) is proportional to \( n \), so that

the % of \( \text{O}_2 \) by volume = mol % of \( \text{O}_2 \)

At the surface: \( P_{\text{O}_2} = x_{\text{O}_2} P_{\text{total}} = (0.20) (1.0 \text{ atm}) = 0.20 \text{ atm} \). This partial pressure must be maintained underwater at \( P_{\text{total}} = 4.0 \text{ atm} \), so that underwater

\[ x_{\text{O}_2} = P_{\text{O}_2} / P_{\text{total}} = (0.20 \text{ atm})/(4.0 \text{ atm}) = 0.050 \]

so that the % of \( \text{O}_2 \) by volume = 5.0%
From the relation \( q = C_p \Delta T = \bar{n}C_p \Delta T \), where \( \bar{C}_p \) is the molar heat capacity at constant pressure, we have for benzene

\[
C_p = \frac{q}{n\Delta T} = \frac{330 \text{ J}}{(24.6 \text{ g}/78.11 \text{ g mol}^{-1})(28.7 ^\circ \text{C} - 21.0 ^\circ \text{C})} = 136 \text{ J mol}^{-1} \text{C}^{-1}
\]

The chemical equations and the standard enthalpies of combustion of 1 mole of fumaric acid and 1 mole of maleic acid are:

\[
\text{fumaric} + 3\text{O}_2 \rightarrow 4\text{CO}_2 + 2\text{H}_2\text{O} \quad \Delta_rH^\circ = -1336.0 \text{ kJ mol}^{-1}
\]
\[
\text{maleic} + 3\text{O}_2 \rightarrow 4\text{CO}_2 + 2\text{H}_2\text{O} \quad \Delta_rH^\circ = -1359.2 \text{ kJ mol}^{-1}
\]

Isomerization, maleic acid \( \rightarrow \) fumaric acid, is the sum of the second reaction and the first in reverse:

\[
\text{4CO}_2 + 2\text{H}_2\text{O} \rightarrow \text{fumaric} + 3\text{O}_2 \quad \Delta_rH^\circ = 1336.0 \text{ kJ mol}^{-1}
\]
\[
\text{maleic} + 3\text{O}_2 \rightarrow \text{4CO}_2 + 2\text{H}_2\text{O} \quad \Delta_rH^\circ = -1359.2 \text{ kJ mol}^{-1}
\]

The enthalpy of isomerization is then

\[
\Delta_rH^\circ = 1336.0 \text{ kJ mol}^{-1} - (-1359.2 \text{ kJ mol}^{-1}) = 23.2 \text{ kJ mol}^{-1}
\]

(a) Determining \( \Delta_rH^\circ \) is easier for the reverse reaction, the combustion of glucose, which can be measured in a calorimeter. Taking the negative of this result gives \( \Delta_rH^\circ \) for the forward reaction.

(b) Use enthalpies of formation to calculate \( \Delta_rH^\circ \),

\[
\Delta_rH^\circ = \Delta_rH^\circ[\text{C}_6\text{H}_{12}\text{O}_6(s)] + 6\Delta_rH^\circ[\text{O}_2(g)] - 6\Delta_rH^\circ[\text{CO}_2(g)] - 6\Delta_rH^\circ[\text{H}_2\text{O}(l)]
\]
\[
= -1274.5 \text{ kJ mol}^{-1} + 6(0 \text{ kJ mol}^{-1}) - 6(-393.5 \text{ kJ mol}^{-1}) - 6(-285.8 \text{ kJ mol}^{-1})
\]
\[
= 2801.3 \text{ kJ mol}^{-1}
\]

(The value for \( \alpha \)-D-glucose is used.) This is the enthalpy change for the reaction forming 1 mole of glucose.

Over a year \( 7.0 \times 10^{14} \) kg of glucose is produced by photosynthesis. This corresponds to

\[
(7.0 \times 10^{14} \text{ kg})(1000 \text{ g}/1 \text{ kg})(1 \text{ mol C}_6\text{H}_{12}\text{O}_6/180.16 \text{ g C}_6\text{H}_{12}\text{O}_6)
\]
\[
= 3.9 \times 10^{15} \text{ mol C}_6\text{H}_{12}\text{O}_6
\]

Then:

\[
\Delta_rH^\circ = (3.9 \times 10^{15} \text{ mol})(2801.3 \text{ kJ mol}^{-1}) = 1.1 \times 10^{19} \text{ kJ}
\]
The fuel value of 1 pound of hamburger is

\[(1 \text{ lb})(454 \text{ g/1 lb})(3.6 \text{ kcal g}^{-1})(4.184 \text{ kJ/1 kcal}) = 6.84 \times 10^3 \text{ kJ}\]

For the vaporization of water at 298K, \(\text{H}_2\text{O (l)} \rightarrow \text{H}_2\text{O (g)}\), \(\Delta_{\text{vap}}H = 44.0 \text{ kJ mol}^{-1}\).
This can be found from 

\[\Delta_{\text{vap}}H = \Delta_{f}H^{o}[\text{H}_2\text{O(g)}] - \Delta_{f}H^{o}[\text{H}_2\text{O(l)}] = -241.8 \text{ kJ mol}^{-1} - (-285.8 \text{ kJ mol}^{-1})\]

Assuming that the entire fuel value of the hamburger is used to vaporize water, it will require that

\[(6.84 \times 10^3 \text{ kJ/44.0 kJ mol}^{-1})(18.02 \text{ g H}_2\text{O/1 mol H}_2\text{O}) = 2.8 \times 10^3 \text{ g H}_2\text{O}\]

be vaporized.