1. Given a second order linear homogeneous differential equation

\[ a_2(x)y'' + a_1(x)y' + a_0(x)y = 0 \]

we know that a fundamental set for this ODE consists of a pair linearly independent solutions \( y_1, y_2 \). But there are times when only one function, call it \( y_1 \), is available and we would like to find a second linearly independent solution. We can find \( y_2 \) using the method of reduction of order.

First, under the necessary assumption the \( a_2(x) \neq 0 \) we rewrite the equation as

\[ y'' + P(x)y' + Q(x)y = 0, \quad P(x) = \frac{a_1(x)}{a_2(x)}, \quad Q(x) = \frac{a_0(x)}{a_2(x)}, \]

Then the method of reduction of order gives a second linearly independent solution as

\[ y_2(x) = u(x)y_1(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)} dx. \]

Given the problem

\[ 25y'' - 20y' + 4y = 0 \]

and a solution \( y_1 = e^{(2x/5)} \). Applying the reduction of order method to this problem we obtain the following

\[ y_1^2(x) = e^{(4x/5)} \]

\[ P(x) = -\frac{4}{5} \]

\[ e^{-\int P(x)dx} = e^{(x/5)} \]

Finally, we arrive at

\[ y_2(x) = u(x)y_1(x) = y_1(x) \int \frac{e^{-\int P(x)dx}}{y_1^2(x)} dx = e^{(4x/5)} \int e^{(x/5)} dx = e^{(6x/5)} \]

So the general solution to \( 25y'' - 20y' + 4y = 0 \) can be written as

\[ y = c_1 y_1 + c_2 y_2 = \ldots \]

\[ y = c_1 e^{(2x/5)} + c_2 x e^{(2x/5)} \]
2. Given the problem 
\[ x^2y'' + 5xy' - 12y = 0 \]
and a solution \( y_1 = x^2 \). Applying the reduction of order method to this problem we obtain the following

\[ y_1^2(x) = \ldots x^4 \]
\[ P(x) = \frac{5}{x} \]
\[ e^{-\int P(x) dx} = \frac{1}{x^5} \]

Finally, we arrive at

\[ y_2(x) = u(x)y_1(x) = y_1(x) \int \frac{e^{-\int P(x) dx}}{y_1^2(x)} \, dx = \ldots \]

So the general solution to \( x^2y'' + 5xy' - 12y = 0 \) can be written as

\[ y = c_1y_1 + c_2y_2 = \ldots \]

\[ e^{-\int \frac{1}{x} \, dx} = e^{-5|ln|x||} = e^{ln|x|^5} = \frac{1}{x^5} \]

\[ y_2(x) = x^2 \int \frac{\frac{1}{x^5}}{x^4} \, dx = x^2 \int x^{-9} \, dx \]
\[ = x^{-8} \left( \frac{x^{-8}}{-8} \right) = -\frac{1}{8} x^{-6} \]

So \( y = c_1 x^2 - \frac{c_2}{8} x^{-6} \)