Relative Extrema: 1. Relative maximum 2. Relative minimum
Let \( f(x) \) be a function.
- 1. The function \( f(x) \) has a relative maximum at \( c \) if the output \( f(c) \) is greater than any other output in some interval around \( c \).
- 2. The function \( f(x) \) has a relative minimum at \( c \) if the output \( f(c) \) is smaller than any other output in some interval around \( c \).

Example:

\[ P(x) \]
\[ \text{Profit in thousands dollars} \]
\[ \text{Number of cellphones sold} \]

D and H are relative maxima and F is a relative minimum.
- Absolute Extrema points not required.

Critical points.

A critical point of a continuous function \( f(x) \) is a point \((c, f(c))\) at which \( f(x) \) is not differentiable, or \( f'(c) = 0 \).

The input value \( c \) is called the critical input for the critical point \((c, f(c))\).

Example: Find the critical points, the maxima, the minima:
1. **Definition Test for Relative Extrema**

Suppose \( c \) is a critical input. Compute the value of \( f(c) \), \( f(c - h) \) and \( f(c + h) \) for a small number \( h \). (For example, \( h \) can be 0.1, or 1, or ...). Then use the definition.

**Example:** Find all extreme points and identify the extreme as a maximum or minimum for function \( g(x) = -3x^2 + 14.1x - 16.2 \)

\[
\begin{array}{c|c|ccc}
\text{Solve } g'(x) = 0 & x & 2 & 2.35 & 3 \\
-6x + 14.1 = 0 & g(x) & 0 & 0.3675 & -0.9 \\
\end{array}
\]

\( x = 2.35 \)

So, \( g(x) \) has a maximum at \( x = 2.35 \)

2. **The First Derivative Test for Relative Extrema**

Suppose \( c \) is a critical input of a continuous function \( f(x) \).
- If \( f'(x) \) changes from positive(+) to negative(−) at \( c \), then \( f(c) \) is relative maximum.
- If \( f'(x) \) changes from negative(−) to positive(+) at \( c \), then \( f(c) \) is relative minimum.
- If \( f'(x) \) does not change sign at \( c \) then \( f(c) \) is not a relative extreme point.

**Example:** Using the first derivative test for Example 1.

\[
\begin{array}{c|c|ccc}
\text{ } & x & 2 & 3 \\
g'(x) & 2.1 & -3.9 \\
\end{array}
\]

\( g'(x) \) change from + to − at \( x = 2.35 \)

So, \( g(2.35) \) is a maximum

**Example:** Find all extreme points and identify the extreme as a maximum or minimum for the function \( f(x) = x^3 + 4x^2 - 16x + 5 \).

\[
\begin{array}{c|c|ccc}
\text{Solve } f'(x) = 0 & x & -4 & \frac{4}{3} \\
3x^2 + 8x - 16 = 0 & f(x) & 60 & 69 & 62 \\
1x & 4 & \frac{4}{3} & 2 \\
3x & -4 & & & \\
(x + 4)(3x - 4) = 0 & f(x) & 19 & -1 & 12 \\
x + 4 = 0 & x & -4 & \frac{4}{3} \\
x = -4 & f(-4) & -3 & & \\
x = \frac{4}{3} & f\left(\frac{4}{3}\right) & & 2 & \\
so \ f(4) is a maximum \ f\left(\frac{4}{3}\right) is a minimum
\end{array}
\]
Example: \( f(x) = 3x^3 - 1.5x^2 - 20x \)

Solve \( f(x) = 0 \)

\[ 9x^2 - 3x - 20 = 0 \]
\[ 3x \cdot 4 \]
\[ 3x - 5 \]

\[ (3x+4)(3x-5) = 0 \]
\[ x = -\frac{4}{3}, \; x = \frac{5}{3} \approx 1.67 \]
\[ \approx 1.33 \]

\[ \text{Definition Test} \]
\[ \text{① } x = -\frac{4}{3} \]

\[ \begin{array}{c|ccc}
   x & -2 & -\frac{4}{3} & -1 \\
   f(x) & 10 & 16.89 & 15.5 \\
\end{array} \]

\[ \text{② } x = \frac{5}{3} \]

\[ \begin{array}{c|ccc}
   x & 1 & \frac{5}{3} & 2 \\
   f(x) & -18.5 & -23.61 & -22 \\
\end{array} \]

\[ \text{First Derive Test} \]
\[ \text{① } x = -\frac{4}{3} \]

\[ \begin{array}{c|ccc}
   x & -2 & -\frac{4}{3} & -1 \\
   f'(x) & 22 & 8 & -8 \\
\end{array} \]

So, \( f(-\frac{4}{3}) \) is a maximum

\[ \text{② } x = \frac{5}{3} \]

\[ \begin{array}{c|ccc}
   x & 1 & \frac{5}{3} & 2 \\
   f'(x) & -14 & -10 & 10 \\
\end{array} \]

So, \( f(\frac{5}{3}) \) is a minimum