1. The General and Specific Antiderivative
   - The general antiderivative (indefinite integral) of \( f(x) \) is
     \[
     \int f(x)\,dx = F(x) + C,
     \]
   where \( F(x) \) is an antiderivative and \( C \) is an arbitrary constant number.
   - When the constant \( C \) is known, \( F(x) + C \) is a specific antiderivative.

\[
\begin{align*}
  \int kdx &= kx + C \\
  \int x^{-1}dx &= \ln|x| + C \\
  \int e^{kx}dx &= \frac{e^{kx}}{k} + C \\
  \int x^n\,dx &= \frac{x^{n+1}}{n+1} + C \\
  \int b^x\,dx &= \frac{b^x}{\ln b} + C \\
  \int e^x\,dx &= e^x + C
\end{align*}
\]

2. Finding a specific anti-derivative
   Example: Write a formula for \( F \), the specific antiderivative of \( f \).

1. \( f(x) = 6x^2 + 16; \quad F(2) = 37. \)
   \[
   \int f(x)\,dx = \frac{6x^3}{3} + 16x + C = 2x^3 + 16x + C
   \]
   \[
   F(2) = 2 \cdot 2^3 + 16 \cdot 2 + C = 37 \\
   48 + C = 37 \\
   C = -11
   \]
   \[
   F(x) = 2x^3 + 16x - 11
   \]

2. \( f(u) = \frac{2}{u} + u; \quad F(1) = 5 \)
   \[
   \int f(u)\,du = 2\ln|u| + \frac{u^2}{2} + C
   \]
   \[
   F(1) = 2\ln(1) + \frac{1^2}{2} + C = 5
   \]
   \[
   C = 4.5
   \]
   \[
   F(1) = 2\ln|1| + \frac{1^2}{2} + 4.5
   \]

3. \( f(x) = 3e^{2x} + 15x^5; \quad F(0) = 8 \)
   \[
   \int f(x)\,dx = \frac{3e^{2x}}{2} + \frac{15x^6}{6} + C
   \]
   \[
   F(0) = \frac{3}{2} + C = 8
   \]
   \[
   C = 6.5
   \]
   \[
   F(x) = \frac{3e^{2x}}{2} + \frac{5x^6}{2} + 6.5
   \]
**Example:** (HW23 in Textbook Page 364)

**Fuel Consumption.** The rate of change of the average annual fuel consumption of passenger vehicles, buses, and trucks from 1970 through 2000 can be modeled as

\[ f(t) = 0.8t - 15.9 \text{ gallons per vehicle per year} \]

where \( t \) is the number of years since 1970. The average annual fuel consumption was 712 gallons per vehicle in 1980. (Source: Based on data from Bureau of Transportation Statistics)

\[ f(0) \approx 712 \]

Q: Write the specific antiderivative giving the average annual fuel consumption.

\[
F(t) = 0.8t^2 - 15.9t + C
\]

\[
F(0) = 0.8 \times \frac{100}{2} - 159 + C = 712
\]

\[ C = 831 \]

\[ F(t) = 0.4t^2 - 15.9t + 831 \text{ gallons/vehicle.} \]

**Example:** (HW21 in Textbook Page 373)

**Investment Growth** An investment worth $1 million in 2005 has been growing at a rate of

\[ f(t) = 0.140 (1.15^t) \text{ million $ per year} \]

where \( t \) is the number of years since 2005.

Q: Calculate how much the investment will have grown between 2005 and 2015 and how much it is projected to grow between 2015 and 2020.

\[
F(t) = \int f(t) \, dt = \int 0.140 (1.15^t) \, dt
\]

\[ = 0.140 \frac{1.15^t}{\ln 1.15} + C \]

\[ F(10) - F(0) \approx 3.051 \text{ million $} \]

\[ F(15) - F(10) \approx 4.098 \text{ million $} \]

**Quiz6Review**

\[ F(0) = 1 \quad \frac{0.14}{\ln 1.15} + C = 1 \]

\[ C = -0.0017 \]

\[ F(t) = 1.0017 (1.15)^t - 0.0017 \]