1. The Slope Graph of a function

The **Slope Graph** is the graph of the new function $f'(x)$.

- If we have the algebra of $f(x)$, we can use the limits definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

computing $f'(x)$, then find the graph precisely.

- If we only have the graph of a function $f(x)$, how to find the information of all the derivatives $f'(x)$? (Estimate)

**Example 1**

**Example 2**

**Example 3**

*Logistic model*
1. **Example** The figure gives the average monthly profit for a cellphone store in the last year corresponding to the number of cellphones sold monthly. Answer the following questions (include Units).

![Graph showing profit in thousands of dollars vs. number of cellphones sold]

**Q1:** Label the positive (+), negative (-), or zero (0) for the (instantaneous) rate of change at

\[ A \quad + \quad B \quad + \quad C \quad + \quad D \quad 0 \quad E \quad - \quad F \quad 0 \quad G \quad + \quad H \quad 0 \]

**Q2:** Draw tangent lines at point C, at point F and at point G.

**Q3:** Label the positive (+), negative (-), or zero (0) for the average rate of change between two points.

\[ A \rightarrow C \quad + \quad D \rightarrow F \quad - \quad C \rightarrow G \quad + \]

**Q4:** When the 20 cellphones sold monthly, what is the profit of the store?

\[ 12 \text{ thousand dollars} \]

**Q5:** When the number of cellphones sold from change from 20 to 80, what is the change, percentage change, and average rate of change?

\[ \text{Change} = f(x_1) - f(x_0) = 32 - 12 = 20 \text{ thousand } \$ \]

\[ \text{Percentage change} = \frac{32 - 12}{12} \times 100\% = 166.67\% \]

\[ \text{ARC} = \frac{32 - 12}{80 - 20} = \frac{20}{60} = 0.333 \text{ thousand } \$ \text{ per cellphone} \]

**Q6:** Are the change, the percentage change, the average rate of change have the same sign (+, -, or 0)? \[ \text{Yes} \]
Q7: Where are the inflection points?

Change of concavity

B.E.G

Q8: Estimate the specific values of number of cellphone sold monthly, for which the derivative of the profit is zero.

35  60  100

Q9: At least how many cellphones the store should sell monthly, if it want to earn money?

11 cellphones

2. Powers and Logarithms (Page 41 in the ClassPacket.)

\[ x^a \cdot x^b = x^{a+b} \quad \frac{x^a}{x^b} = x^{a-b} \quad (x^a)^b = x^{ab} \quad x^{-a} = \frac{1}{x^a} \quad \sqrt{x} = x^{\frac{1}{2}} \quad \sqrt[n]{x^a} = x^{\frac{a}{n}} \]

\[ \ln(e^k) = k \quad e^{\ln(k)} = k \quad \ln(ab) = \ln(a) + \ln(b) \quad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) \quad \ln(a^k) = k \ln(a) \]

\[ \ln(e) = 1 \quad \text{and} \quad \ln(1) = 0. \]

The graphs of \( y = e^x \) and \( y = \ln(x) \) are mirror reflections along the line \( y = x \).
The following graph shows the temperature in Boston last Friday and Saturday, in degrees Fahrenheit.

1. Between noon and midnight September 16, the percentage change in temperature was:
   positive ________ negative ________ zero ________ (circle one)
2. Around 10 am September 17, the instantaneous rate of change of temperature was:
   positive ________ negative ________ zero ________ (circle one)
3. At 6 pm September 16, the temperature was:
   increasing ________ decreasing ________ neither ________ (circle one)
4. Estimate and list the specific times for which the derivative of the temperature is zero.
   Step 16: 6am, 2pm
   Step 17: 4am, 1pm
5. Estimate the temperature at 6 am September 17. Give units.
   53 °F
6. List the approximate location of two inflection points.
   6am 9pm • Step 16
   9am Step 17
Examples: Simplify the following formulas.

1. \(5(x^{-3})^3 - 8 \left(\frac{1}{x^4}\right)^{-2} + \frac{1}{3x^2} = 5x^{-9} - 8x^8 + \frac{1}{3}x^{-2}\)

2. \(3\sqrt{x^3} - 4\sqrt[3]{x^2} - \frac{x}{2x^4} = 3x^{\frac{2}{3}} - 4x^{\frac{2}{3}} - \frac{1}{2}x\)^5

3. \(4\sqrt[5]{x^4} + \frac{x}{\sqrt[5]{x}} = 4x^{\frac{4}{5}} + x^{\frac{3}{5}}\)

4. \(-x^6 - x(3x^{-3} - x^5) = -3x^2\)

5. \(\frac{x^2 - 34x^5 - 12}{2x^4} = \frac{1}{2}x^{-2} - 17x + 6x^{-4}\)

6. \(\left(\sqrt{x^2} - 3 \left(\frac{2}{x}\right)^{-3} + \frac{x}{3x^{-2}}\right) \ln(1) = 0\)

7. \(\ln(e^{-x^2}) + e^{\ln(x^2)} = -x^2 + x^2 = 0\)

8. \(3\sqrt[6]{x^4} + \frac{x}{\sqrt[6]{x}} = 3x^\frac{2}{3} + x^{\frac{3}{6}}\)

9. \(1 - (3x - 1)(x + 3) - (x + 2)^2 = -4x^2 - 12x\)

10. \(\frac{32x^{-3} - 14x^2 - 24x^{-1}}{4x^{-2}} = 8x^2 - \frac{7}{2}x^4 - 6x\)

Ex: Page 11 in Class packet.