Let $\sum_{n=1}^{\infty} a_n$ be a series.

We consider a new series of absolute values:

$$\sum_{n=1}^{\infty} |a_n| = |a_1| + |a_2| + |a_3| + |a_4| + |a_5| + \cdots$$

**Definition**

A series $\sum_{n=1}^{\infty} a_n$ is called **absolutely convergent** if the series of absolute values $\sum_{n=1}^{\infty} |a_n|$ is convergent.
Theorem

If a series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then it is convergent.

Definition

A series $\sum_{n=1}^{\infty} a_n$ is called **conditionally convergent** if it is convergent but not absolutely convergent.
The Ratio Test

The following test is very useful and powerful.

Theorem (The Ratio Test)

Given a series $\sum a_n$, and suppose

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L.$$

(1.) If $L < 1$, then the series $\sum a_n$ is absolutely convergent.

(2.) If $L > 1$ or $L = \infty$, then the series $\sum a_n$ is divergent.

(3.) If $L = 1$, then the Ratio Test is inconclusive.
The Root Test*

The following test is convenient to apply when \( n \)-th powers occur.

**Theorem (The Root Test)**

Given a series \( \sum a_n \), and suppose

\[
\lim_{n \to \infty} |a_n|^{1/n} = L.
\]

(1.) If \( L < 1 \), then the series \( \sum a_n \) is absolutely convergent.
(2.) If \( L > 1 \) or \( L = \infty \), then the series \( \sum a_n \) is divergent.
(3.) If \( L = 1 \), then the Ratio Test is inconclusive.