§9.1 Modeling with Differential Equations

- We have an introduction about differential equations in this chapter. There will be a class **Math 285**–Differential Equations after the class **Math 283**–Calculus 3.

- To solve some real world problems, we need to set up an equation. Sometimes, we need differential equations.

- A **differential equation** is an equation containing an *unknown function* $f(x)$ (or denoted by $y$) and some of its *derivatives* $y'$, $y''$, ....

**Example (1)**

$$y' = -2x.$$
A differential equation often has (infinitely) many solutions.

Solving an equation is hard in general, but verifying whether or not a function \( f(x) \) is a solution is easy.

### Example (3)

One model for the growth of a population

\[
\frac{dP}{dt} = kP
\]

where \( k \) is a constant number, \( t \) is the time (independent variable) and \( P \) is the number of individuals in the population (dependent variable).

- When we apply differential equations to real world problems, we are interested in finding a particular solution satisfying a condition of the form \( y(t_0) = y_0 \), which is called an **initial condition**.
- This kind differential equation with initial condition is called **initial-value problem**.
Many populations start by increasing in an exponential model, however, the populations decrease when they approach its **carrying capacity** $M$.

**Example 7.** [Pierre-Francois Verhulst, 1840s] The world population growth is modeled by the differential equation

$$
\frac{dP}{dt} = kP \left(1 - \frac{P}{M}\right)
$$

where $M$ is the carrying capacity.

Let us see what can we obtain from this model.
\[
\frac{dP}{dt} = kP \left(1 - \frac{P}{M}\right)
\]

1. The constant functions \( P(t) = 0 \) and \( P(t) = M \) are solutions for the differential equation, which are called \textit{equilibrium solutions}.

2. If the initial population \( P(0) < M \), then the right side of the equation is positive, hence \( \frac{dP}{dt} > 0 \) and the population increases.

3. If the initial population \( P(0) > M \), then the right side of the equation is negative, hence \( \frac{dP}{dt} < 0 \) and the population decreases.