§9.2 Slope Fields and Euler’s Method

1. Graphical approach (Slope fields, or direction fields)

If \( y = f(x) \) is a solution for the differential equation \( \frac{dy}{dx} = F(x, y) \), then the slope of curve \( y = f(x) \) at \((x, y)\) is \( F(x, y) \).

At each point \((x, y)\) draw a line segment with slope \( F(x, y) \). The solution is the curve tangent at this point.

Example 1. \( \frac{dy}{dx} = x - y \) (Use slope fields).
2. Numerical approach (Euler’s Method)

Find approximating solution for the differential equation \( \frac{dy}{dx} = F(x, y) \) with initial value \( y(x_0) = y_0 \) using Euler’s Method with step size \( h \):

(1). Set step size \( h \); (the smaller \( h \), the better estimation.)
(2). Start with point \((x_0, y_0)\);
(3). Define a sequence \( x_n = x_{n-1} + h \);
(4). Then \( y_n \) is computed by the sequence

\[
y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})
\]
Example 2. Use Euler’s method with step size \( h = 0.5 \) to solve \( \frac{dy}{dx} = x - y \) with initial value \( y(0) = 1 \).