1. About the Syllabus:
Read the Syllabus carefully!

2. Review some backgrounds:

- **Numbers:**
  Natural numbers \( \mathbb{N} \)
  Integers \( \mathbb{Z} \)
  Rational numbers \( \mathbb{Q} \)
  Real numbers \( \mathbb{R} \)
  Complex numbers \( \mathbb{C} \)

- **Functions:**

  Single variable functions:
  
  \[
  f(x) = 2x + 3; \quad f(x) = x^2 + 8; \quad f(x) = 2e^x + 3; \quad f(x) = 2\sin(x); \\
  f(x) = \ln(x) + 3x - 2; \quad \ldots
  \]

3. Student Learning Outcomes of Calculus 1:
Upon completion of this course, students will be able to:

1. Demonstrate an understanding of concepts and the terminology of **limits** through applications and examples.

2. Compute the **derivative** of a function using the definition, rules of differentiation, slopes of tangent lines and describe it as rate of change in natural and physical phenomena.

3. Compute basic **integrals** using Riemann sums as well as the Fundamental Theorem of Calculus.
5.1. Area and Estimating with Finite (Riemann) Sums

\[ \Delta x = \frac{b-a}{4} \]

\[ \text{Area } \approx \text{ sum of Area of the 4 rectangles.} \]

\[ = f(x_1^*) \Delta x + f(x_2^*) \Delta x + f(x_3^*) \Delta x + f(x_4^*) \Delta x \]

5.2. The Definite Integral

Let \( f(x) \) be a continuous function defined on the interval \([a, b]\). The definite integral (accumulated change) of \( f(x) \) from \( a \) to \( b \) is

\[ \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x. \]

\[ \Delta x = \frac{b-a}{n} \]

Some properties:

\[ \int_{a}^{b} f(x) \pm g(x) \, dx = \int_{a}^{b} f(x) \, dx \pm \int_{a}^{b} g(x) \, dx \]

\[ \int_{a}^{b} k f(x) \, dx = k \int_{a}^{b} f(x) \, dx \]

\[ \int_{a}^{b} f(x) \, dx = \int_{a}^{c} f(x) \, dx + \int_{c}^{b} f(x) \, dx \]

\[ \int_{a}^{b} f(x) \, dx = -\int_{b}^{a} f(x) \, dx \]
5.3. The Fundamental Theorem of Calculus.

**Theorem 1.** If $f(x)$ is a continuous function on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$
\int_a^b f(x)\,dx = F(x)\bigg|_a^b = F(b) - F(a).
$$

**Theorem 2.** If $f(x)$ is a continuous function on the interval $[a, b]$, then

$$
g(x) = \int_a^x f(t)\,dt
$$

is continuous and $g'(x) = f(x)$.
## Review of some formulas from Calculus 1

### Derivatives:

<table>
<thead>
<tr>
<th>Function $f(x)$</th>
<th>Derivative $f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^n$</td>
<td>$n \cdot x^{n-1}$</td>
</tr>
<tr>
<td>$b^x$</td>
<td>$(\ln b) \cdot b^x$</td>
</tr>
<tr>
<td>$e^x$</td>
<td>$e^x$</td>
</tr>
<tr>
<td>$\ln(x)$</td>
<td>$\frac{1}{x}$</td>
</tr>
<tr>
<td>$\sin(kx)$</td>
<td>$k \cos(kx)$</td>
</tr>
<tr>
<td>$\cos(kx)$</td>
<td>$-k \sin(kx)$</td>
</tr>
</tbody>
</table>

### Indefinite integral:

<table>
<thead>
<tr>
<th>Function $f(x)$</th>
<th>$\int f(x) , dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^n$, $(n \neq -1)$</td>
<td>$\frac{x^{n+1}}{n+1} + C$</td>
</tr>
<tr>
<td>$\frac{1}{x}$</td>
<td>$\ln</td>
</tr>
<tr>
<td>$b^x$</td>
<td>$\frac{b^x}{\ln(b)} + C$</td>
</tr>
<tr>
<td>$e^{kx}$</td>
<td>$\frac{e^{kx}}{k} + C$</td>
</tr>
<tr>
<td>$\sin(kx)$</td>
<td>$-\frac{1}{k} \cos(kx) + C$</td>
</tr>
<tr>
<td>$\cos(kx)$</td>
<td>$\frac{1}{k} \sin(kx) + C$</td>
</tr>
</tbody>
</table>

**Chain Rule:** $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

**Product Rule:** $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
Evaluate the definite integrals:

**Example 1.** Find \( \int_{1}^{2} \frac{4x^5 + 6}{x^2} \, dx \)

\[
\int_{1}^{2} \frac{4x^5 + 6}{x^2} \, dx = \int_{1}^{2} 4x^3 + 6x^{-2} \, dx \\
= \left[ x^4 - 6x^{-1} \right]_{1}^{2} \\
= (2^4 - 6(\frac{1}{2})) - (1 - 6) = 13 + 5 = 18
\]

**Example 2.** Find \( \int_{0}^{\pi} \sin 3x \, dx \)

\[
\int_{0}^{\pi} \sin 3x \, dx = -\frac{1}{3} \cos 3x \bigg|_{0}^{\pi} \\
= -\frac{1}{3} (\cos 3\pi - \cos 0) \\
= \frac{2}{3}
\]

**Example 3.** Find \( \int_{0}^{1} e^{2x} \, dx \)

\[
\int_{0}^{1} e^{2x} \, dx = \frac{1}{2} e^{2x} \bigg|_{0}^{1} = \frac{1}{2} e^2 - \frac{1}{2}
\]
Example 4 Show that \( \ln x = \int_1^x \frac{1}{t} \, dt \)

\[
\int_1^x \frac{1}{t} \, dt = \ln t \bigg|_1^x = \ln x - \ln 1 = \ln x
\]

Find the derivative of the following functions.

Example 5. \( f(x) = \int_3^{2x^3-1} \sin(2t) \, dt \)

Suppose \( F(t) \) is an antiderivative of \( \sin 2t \), then \( F'(t) = \sin 2t \)

\[
f(x) = F(2x^3-1) - F(3)
\]

\[
f'(x) = F'(2x^3-1) \cdot 6x^2 = \left( \sin 2(2x^3-1) \right) \cdot 6x^2
\]

Example 6. \( f(x) = \int_1^{x^2} e^{t^2} \, dt \)

Suppose \( F(t) \) is an antiderivative of \( e^{t^2} \), then \( F'(t) = e^{t^2} \)

Then \( f(x) = F(x^2) - F(1) \)

\[
f'(x) = F'(x^2) \cdot 2x - 0
\]

\[
= e^{(x^2)^2} \cdot 2x = e^{x^4} \cdot 2x
\]