Example 1. \( y' = -2x \).

Example 2. Verify that \( y = ce^{2x} \) (\( c \) is any real number) are solutions for the differential equation \( y' = 2y \).

Example 3. One model for the growth of a population

\[
\frac{dP}{dt} = kP
\]

where \( k \) is a constant number, \( t \) is the time (independent variable) and \( P \) is the number of individuals in the population (dependent variable).

Example 4. Verify that \( y = a\sin 3x + b\cos 3x \) (where, \( a, b \) are any real numbers) are solutions for the differential equation \( y'' = -9y \).

Example 5. (1) For what values of \( r \) does the function \( y = e^{rx} \) a solution for \( y'' - y' - 2y = 0 \)?

(2) If \( r_1 \) and \( r_2 \) are the values of \( r \) that you found in part (1), show that every member of the family of functions \( y = ae^{r_1x} + be^{r_2x} \) is a solution for \( y'' - y' - 2y = 0 \).

Example 6. (1) For what values of \( r \) does the function \( y = e^{rx} \) a solution for \( y' = 3x^2y \)?

(2) Show that every member of the family of functions \( y = ke^{x^3} \) is a solution for \( y' = 3x^2y \). Here, \( k \) is any real number.

(3) Find a solution of differential equation \( y' = 3x^2y \) with initial condition \( y(0) = 2 \).

Example 7. [Pierre-Francois Verhulst, 1840s] The world population growth is modeled by the differential equation

\[
\frac{dP}{dt} = kP \left( 1 - \frac{P}{M} \right)
\]

where \( M \) is the carrying capacity.

Let us see what can we obtain from this model.