To receive full credit for a problem you must show all necessary work.

1. (Extreme Values) Let $f(x,y) = x^3 + y^2 - 3x^2 - 4y + 10$.

(1)(4 points). Find all critical points of $f(x,y)$.

(2)(6 points). Use the second derivative test to determine the local maximum, the local minimum, and saddle point of the function $f(x,y)$. 

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University of Nevada, Reno
MATH 283 (Calculus 3) Fall 2018
Final Exam- Sample Test (100 points)

Instructor: He Wang                     Student Name: ______________________
2. (Double Integral) Let $D$ the region bounded by the line $y = x$ and the parabola $y = x^3$ in the first quadrant. Answer the following questions.

(1). (4 points). Find the intersection points and sketch the region $D$.

(2). (5 points). Calculate the double integral $\int\int_D 4xy + 2x^2 \, dA$.

(3). (1 point). What is the volume of the solid that lies below the surface $z = 4xy + 2x^2$ and above the region $D$?
3. **(Triple Integral)** (12 points). Evaluate \( \iiint_B (x^2 + y^2 + z^2)^2 \, dV \), where \( B \) is the ball of radius 1 centered at the origin. (Hint: using spherical coordinates)
4. **(Line Integral)** (12 points).
Find the work done by a force field \( \vec{F}(x, y) = \langle 2y^3, -2xy^2 \rangle \) moving a particle along the curve \( C \) given by \( \vec{r}(t) = \langle \sin t, \cos t \rangle \), when \( 0 \leq t \leq \pi/2 \).

Hint: \( \cos 2t = 2\cos^2 t - 1 \).
5. (Green’s theorem)(12 points). Evaluate $\int_C (2y + e^x \cos x) \, dx + (x^2 - \ln(y^2 + 3)) \, dy$, where $C$ is the circle $x^2 + y^2 = 9$ with positive direction.
6. **(Surface Integral)** (12 points). Compute the surface integral \( \iint_S 3yz \, dS \), where \( S \) is the sphere defined by \( x^2 + y^2 + z^2 = 4 \) in the first octant.
7. **(Stokes’ Theorem or Divergence Theorem)** (12 points). Compute the integral \[ \iint_S \text{curl} \vec{F} \cdot d\vec{S}, \]
where \( \vec{F} = \langle -yz, xz, x^2y \rangle \) and \( S \) is the sphere \( x^2 + y^2 + z^2 = 4 \) above \( z = 1 \).
8. (4 points) Let \( \vec{F} = (2xy) \vec{i} + (xyz) \vec{j} + (x^2) \vec{k} \).

(1) \text{div}(\vec{F}) =

(2) \text{curl}(\vec{F}) =

9. (4 points) Suppose the surface is given by parametric equations \( x = 2u, y = uv, z = v^2 \).

(1) Find the normal vector of the surface at \((2, 1, 1)\).

Answers:________________________

(2) Find the tangent plane to the surface at \((2, 1, 1)\).

Answers:________________________

10. (2 points) Suppose we know that \( \vec{a} \times \vec{b} = \vec{i} + 2\vec{j} + \vec{k} \) and \( \vec{a} \cdot \vec{b} = \sqrt{2} \). What is the area of the parallelogram spanned by \( \vec{a} \) and \( \vec{b} \)?

Answers:________________________

11. (2 points) Let \( f(x, y) = 4x + x^2y + y^2/3 \) and \( \vec{F}(x, y) = \nabla f = \langle 4 + 2xy, x^2 + y^2 \rangle \). Evaluate the line integral \( \int_C \vec{F} \cdot d\vec{r} \), where \( C \) is the curve defined by \( \vec{r}(t) = \langle e^t \cos t, e^t \sin t \rangle \) for \( 0 \leq t \leq \pi/2 \).

Answers:________________________

12. True or False. (8 points)

(1) If \( \vec{a} \) and \( \vec{b} \) orthogonal, then \( \vec{a} \cdot \vec{b} = 0 \).  \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}

(2) The cross product of two unit vectors is a unit vector. \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}

(3) Line integrals of any vector fields are independent of path. \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}

(4) If \( \vec{a} \) and \( \vec{b} \) parallel, then \( \vec{a} \times \vec{b} = \vec{0} \). \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}

(5) If \( \vec{a}, \vec{b} \) and \( \vec{c} \) coplanar, then \( \vec{b} \cdot (\vec{a} \times \vec{c}) = 0 \). \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}

(6) If \( \vec{a} \cdot \vec{b} = 0 \), then \( \vec{a} = \vec{0} \) or \( \vec{b} = \vec{0} \). \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}

(7) The area of the region \( D \) enclosed by a smooth closed curve \( C \)

\[ \text{equals the line integral} \int_C \frac{1}{3}xdy + \frac{2}{3}ydx \] \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}

(8) \( \vec{F}(x, y) = \langle e^y + \cos y, xe^y - x \sin y \rangle \) is conservative. \hspace{2cm} \text{(A) True} \hspace{2cm} \text{(B) False}