1. (5 points) Let \( \vec{F}(x, y) = (e^y + 2xy^2)i + (xe^y + 2x^2y)j \).

(a) Determine whether or not \( \vec{F} \) is conservative.

\[
\begin{align*}
\frac{\partial P}{\partial y} &= e^y + 4xy \\
\frac{\partial Q}{\partial x} &= e^y + 4xy
\end{align*}
\]

So, \( \vec{F} \) is conservative.

\[\text{continuous}\]

(b) Find a function \( f \) such that \( \nabla f = \vec{F} \).

\[
\begin{align*}
f_x &= e^y + 2xy^2 \\
f_y &= xe^y + 2x^2y
\end{align*}
\]

\[
\Rightarrow f = xe^y + x^2y^2 + g(y)
\]

\[
\begin{align*}
\frac{\partial g'}{\partial y} &= 0 \\
\Rightarrow g' &= K \text{ constant number}
\end{align*}
\]

\[
\Rightarrow f = xe^y + x^2y^2 + K.
\]

(3) Use part (2) to evaluate \( \int_C \vec{F} \cdot d\vec{r} \) where \( C \) is a smooth curve from \( (1, 1) \) to \( (4, 0) \).

\[
\int_C \vec{F} \cdot d\vec{r} = f(4, 0) - f(1, 1) = 4 - (e + 1) = 3 - e
\]
2. (5 points) Evaluate \( \int_C \left(3y + e^y \sin x\right) \, dx + \left(8x - \ln(y^3 + 2)\right) \, dy \), where \( C \) is the circle \( x^2 + y^2 = 4 \) with positive direction.

\[
\oint_C P \, dx + Q \, dy
= \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA
= \iint_D (8 - 3) \, dA
= \int_0^{2\pi} \int_0^2 5r \, dr \, d\theta
= \int_0^{2\pi} d\theta \int_0^2 5r \, dr
= 2\pi \cdot \left. \frac{5r^2}{2} \right|_0^2
= 20\pi
\]

3. True or False: (1 bonus point)

(1) Line integrals of any vector fields are independent of path. ...... (A) True (B) False

(2) The area of the region \( D \) enclosed by a smooth closed curve \( C \) equals the line integral \( \oint_C \frac{1}{3} x \, dy - \frac{2}{3} y \, dx \) ...... (A) True (B) False