1. (4 points). Let \( R \) be the region \( R = \{ (x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1 \text{ and } 1 \leq y \leq 2 \} \). Calculate the double integral \( \iint_R 6xy^2 \, dA \)

\[
\iint_R 6xy^2 \, dA = \int_1^2 \left[ \int_0^1 6xy^2 \, dx \right] \, dy
\]

\[
= \int_1^2 3x^2y^2 \bigg|_{x=0}^{x=1} \, dy
\]

\[
= \int_1^2 3y^2 \, dy = y^3 \bigg|_1^2 = 8 - 1 = 7
\]

2. (6 points). Let \( D \) the region bounded by the line \( y = x + 1 \) and the parabola \( y = x^2 - 5 \). Sketch the region \( D \), and calculate the double integral \( \iint_D 2x \, dA \).

\[
\int_{-2}^{3} \int_{x^2-5}^{x+1} 2x \, dy \, dx
\]

\[
= \int_{-2}^{3} 2xy \bigg|_{y=x^2-5}^{y=x+1} \, dx
\]

\[
= \int_{-2}^{3} 2x(x+1) - 2x(x^2-5) \, dx
\]

\[
= \int_{-2}^{3} 2x^2 + 2x^2 + 12x \, dx
\]

\[
= \left[ -\frac{x^4}{2} + \frac{2x^3}{3} + 6x^2 \right]_{-2}^{3}
\]

\[
= \left( -\frac{3^4}{2} + \frac{2(3)^3}{3} + 6(3)^2 \right) - \left( -\frac{2^4}{2} + \frac{2(2)^3}{3} + 6(2)^2 \right)
\]

\[
= \left( -\frac{81}{2} + \frac{54}{3} + 54 \right) - \left( -\frac{16}{2} + \frac{32}{3} + 24 \right)
\]

\[
= 7
\]