Example 1. (Gravitational Field) Let \( \vec{x} = (x, y, z) \in \mathbb{R}^3 \). The gravitational force acting on the object at \( \vec{x} \) is
\[
\vec{F}(\vec{x}) = -\frac{mMG}{|\vec{x}|^3} \vec{x}
\]
m and \( M \) are masses of the two objects. \( G = 6.67408 \times 10^{-11} \) is the universal Gravitational constant. The Gravitational Field is a conservative vector field, \( \vec{F} = \nabla f \), for
\[
f(x, y, z) = \frac{mMG}{\sqrt{x^2 + y^2 + z^2}}
\]

**Problem:** Find the work done by \( \vec{F} \) in moving a particle with mass \( m \) from point \((1, 0, 0)\) to \((1, 2, 9)\).

Example 2. Determine whether or not each of the vector field is conservative.

1. \( \vec{F}(x, y) = (2x + y)\vec{i} + (x + 2y)\vec{j} \)
2. \( \vec{F}(x, y) = (2x - y, x + 1) \)
3. \( \vec{F}(x, y) = (4 + 2xy, x^2 + y^2) \)

Example 3. Let \( \vec{F}(x, y) = (4 + 2xy, x^2 + y^2) \).

1. Find a function \( f \) such that \( \nabla f = \vec{F} \).
2. Evaluate the line integral \( \int_C \vec{F} \cdot d\vec{r} \), where \( C \) is the curve defined by \( \vec{r}(t) = (e^t \cos t, e^t \sin t) \) for \( 0 \leq t \leq \pi/2 \).

Example 4. Let \( \vec{F}(x, y) = (e^y + \cos y, xe^y - x \sin y) \).

1. Determine whether or not \( \vec{F} \) is conservative.
2. Find a function \( f \) such that \( \nabla f = \vec{F} \).
3. Use part (2) to evaluate \( \int_C \nabla f \cdot d\vec{r} \) where \( C \) is a curve from \((1, 0)\) to \((0, 3)\).

Example 5. Let \( \vec{F}(x, y) = (6x^2y + 2x \ln y)\vec{i} + (2x^3 + \frac{x^2}{y})\vec{j} \).

1. Determine whether or not \( \vec{F} \) is conservative.
2. Find a function \( f \) such that \( \nabla f = \vec{F} \).
3. Use part (2) to evaluate \( \int_C \nabla f \cdot d\vec{r} \) where \( C \) is a curve from \((1, 1)\) to \((0, 4)\).

Example 6. Let \( \vec{F}(x, y, z) = (yz, xz, xy + e^z) \).

1. Find a function \( f \) such that \( \nabla f = \vec{F} \).
2. Use part (1) to evaluate \( \int_C \nabla f \cdot d\vec{r} \) where \( C \) is a line from \((1, 2, 0)\) to \((0, 3, 1)\).