Theorem 1 (The inverse matrix theorem). Let $A$ be an $n \times n$ matrix. Then the next 18 statements are all equivalent (that is, they are either all true or all false).

(1) $A$ is invertible.
(2) There exists an $n \times n$ matrix $C$ such that $C \cdot A = I_n$.
(3) There exists an $n \times n$ matrix $D$ such that $A \cdot D = I_n$.
(4) $A^T$ is an invertible matrix.
(5) $A$ is row-equivalent to $I_n$.
(6) $A$ has $n$ pivot positions.
(7) The columns of $A$ form a linearly independent set.
(8) The matrix equation $A\vec{x} = \vec{0}$ only has the trivial solution.
(9) The equation $A\vec{x} = \vec{b}$ has a solution for each $\vec{b} \in \mathbb{R}^n$.
(10) The linear transformation $T_A(\vec{x}) = A\vec{x}$ is one-to-one.
(11) The span of the columns of $A$ is $\mathbb{R}^n$.
(12) The image of $T_A$ is $\mathbb{R}^n$.

§3
(13.) $\det(A) \neq 0$.

§4
(14.) The columns of $A$ form a basis for $\mathbb{R}^n$.
(15.) $\text{Col } A = \mathbb{R}^n$.
(16.) $\dim(\text{Col } A) = n$.
(17.) $\text{rank } A = n$.
(18.) $\text{Nul } A = \{\vec{0}\}$.
(19.) $\dim(\text{Nul } A) = 0$. 