1. A quadrilateral $\square ABCD$ is called a *Saccheri quadrilateral* if $AB \cong CD$ and if the angles $\angle B$ and $\angle C$ are right angles, see figure 1. Show that in a Saccheri quadrilateral the congruency relation $\angle A \cong \angle D$ holds.

![Figure 1. A Saccheri quadrilateral.](image1)

2. Show that any model of neutral geometry has to have infinitely many points.

3. Show that in neutral geometry there is an equivalence between the two statements:
   (a) Hilbert’s parallel postulate holds.
   (b) The angle sum in every triangle is $180^\circ$.

4. Prove that in neutral geometry, a line cannot be contained in the interior of an angle.

5. Prove that in neutral geometry every angle has a unique bisector. This is the content of proposition 4.4a.

6. Prove that in neutral geometry, the diagonals of a convex quadrilateral intersect.

7. Prove that nonisosceles triangles exist? Which axioms of neutral geometry do you need to use to prove this existence?

8. Let $\triangle ABC$ be a right triangle with the right angle at $C$ and with the angle $\angle A$ being acute. Create a new right triangle $\triangle AB'C'$ whose angle $\angle A$ coincides with $\angle A$ of the first triangle but whose hypotenuse has been double in length i.e. $AB' = 2 \cdot AB$, see figure 2.

![Figure 2](image2)

(a) Show that $B'C' \geq 2 \cdot BC$.

(b) Show that $AC' \leq 2 \cdot AC$. 
9. A parallelogram is a quadrilateral whose opposite sides lie on parallel lines. Given a quadrilateral $\square ABCD$ show that in neutral geometry
(a) Opposite sides of the parallelogram are congruent.
(b) Show that a parallelogram is a rectangle if and only if its diagonals are congruent.

10. Prove the converse of Euclid V in neutral geometry. That is, in the context of neutral geometry show that if $\ell$ and $\ell'$ two lines which intersect at $A$ and $t$ is a common transversal for $\ell$ and $\ell'$, then the angle sum of the two interior angles of $t$ which lie on the same side of $t$ as $A$ (angles $\angle 1$ and $\angle 2$ in figure 3), have measures which add up to less than $180^\circ$.

![Figure 3](image)

11. Show that in Euclidean geometry (i.e. neutral geometry + Hilbert’s parallel postulate) for every triple of positive numbers $\alpha, \beta, \lambda$ with $\alpha + \beta < 180^\circ$ there is a triangle $\triangle ABC$ with

$$
(\angle A)^\circ = \alpha \quad (\angle B)^\circ \cong \beta \quad AB = \lambda
$$

Is it possible to find such triangles in neutral geometry?