1. Let $X$ be a topological space and $A$ a subset of $X$. Show that

$$X - A = X - \text{Int}(A) \quad \text{and} \quad \text{Int}(X - A) = X - A$$

Use these equalities to show that $\partial A = \partial(X - A)$.

2. Let $(X, <)$ be a simply ordered set and let $T_\sigma$ be the order topology on $X$. Show that with respect to the order topology the inclusion $\langle a, b \rangle \subseteq [a, b]$ holds for any two elements $a, b \in X$ with $a < b$. Under what conditions is the equality $\langle a, b \rangle = [a, b]$ true?

3. For subsets $A \subseteq X$ and $B \subseteq Y$ of the topological spaces $X$ and $Y$, show that $A \times B = \overline{A} \times \overline{B}$ in the product topology on $X \times Y$.

4. Show that $X$ is a Hausdorff space if and only if the diagonal $\Delta = \{(x, x) \mid x \in X \}$ is a closed set in $X \times X$ (equipped with the product topology).