Topology - Homework assignment 2
[Due Wednesday, February 18th]

Instructions: If you are taking the course as Math 440, you only need to complete problems 1–3. If you are enrolled in the course as Math 640, you need to turn in all 4 problems.

1. Let $X$ be a set with at least two points and let $\mathcal{T}_X$ be the indiscrete topology on $X$. Describe the set of continuous functions $f : X \to \mathbb{R}$ where $\mathbb{R}$ is equipped with the Euclidean topology $\mathcal{T}_{Eu}$.

2. Consider $\mathbb{R}$ equipped with the Euclidean topology $\mathcal{T}_{Eu}$ and let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Let $A$ be the subset of $\mathbb{R}$ defined as $A = \{x \in \mathbb{R} \mid f(x) = x\}$ (this set is called the fixed point set of $f$). Show that $A$ is a closed set.

3. Let $\mathbb{R}$ be given the Euclidean topology and let $(a, b) \subset \mathbb{R}$ be equipped with the relative Euclidean topology. Show that $\mathbb{R}$ and $(a, b)$ are homeomorphic topological spaces by finding a concrete homeomorphism $f : \mathbb{R} \to (a, b)$.

4. Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be two topological spaces. Show that a function $f : X \to Y$ is continuous if and only if $f^{-1}(\text{Int}(B)) \subset \text{Int}(f^{-1}(B))$ for every subset $B \subset Y$. 