Math 182 summary notes from lecture 9/15/2010. The basic concepts discussed in yesterday’s lecture are fundamental to our understanding of all of Chapter 8. These notes are intended to summarize the discussion.

If you start with a sequence \( \{a_n\} = \{a_1, a_2, a_3, \ldots\} \) you can form a new sequence \( \{S_n\} = \{S_1, S_2, S_3, \ldots\} \) where \( S_1 = a_1 \); \( S_2 = a_1 + a_2 \); \( S_3 = a_1 + a_2 + a_3 \); and in general, \( S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + \ldots + a_n. \)

This sequence \( \{S_n\} = \{S_1, S_2, S_3, \ldots\} \) is called an infinite series and is usually written as \( \sum a_n. \)

\( S_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + \ldots + a_n \) is called the \( n^{th} \) partial sum of the series.

The fact that there are actually two sequences sometimes causes confusion. There’s \( \{a_n\} \) and then there’s the series \( \sum a_n, \) which is actually another sequence, \( \{S_n\} = \{S_1, S_2, S_3, \ldots\}, \) called the sequence of partial sums of the series. It is obtained by adding the terms of the original sequence \( \{a_n\}. \)

If the sequence \( \{S_n\} \) converges then its limit \( \lim_{n \to \infty} S_n \) is called the sum of the series and is written \( \sum_{n=1}^{\infty} a_n. \) It is possible for the sequence \( \{a_n\} \) to converge while the series \( \sum a_n \) diverges. In particular this is the case when \( a_n = \frac{1}{n}. \) The sequence \( \frac{1}{n} \) converges to 0 but the series \( \sum \frac{1}{n} \) diverges.

In the special case where \( a_n = r^n \) the resulting series is called a geometric series.

- \( \sum_{n=0}^{\infty} ar^n = a \sum_{n=0}^{\infty} r^n = \frac{a}{1-r} \) if \( |r| < 1. \)
- \( \sum_{n=1}^{\infty} ar^{n-1} = a \sum_{n=1}^{\infty} r^{n-1} = \frac{a}{1-r} \) if \( |r| < 1. \)
- Let \( a = r \) in the above and you get \( \sum_{n=1}^{\infty} r^n = \frac{r}{1-r} \) if \( |r| < 1. \)

If \( |r| \geq 1 \) the series diverges.

As mentioned above, it is possible for the sequence \( \{a_n\} \) to converge while the series \( \sum a_n \) diverges. Here are two examples.

The sequence \( \{0.5^n\} \) converges to zero, which can be written as \( 0.5^n \to 0 \) or as \( \lim_{n \to \infty} 0.5^n = 0. \) The resulting geometric series also converges and \( \sum_{n=1}^{\infty} 0.5^n = \frac{0.5}{1-0.5} = 1. \)

Now consider the sequence \( \{a_n\} \) where \( a_n = 2 \) for every \( n. \) This sequence is constant and therefore converges to 2, which can be written as \( a_n \to 2 \) or as \( \lim_{n \to \infty} a_n = 2. \) However, the resulting geometric series diverges. \( \sum_{n=1}^{\infty} a_n = \infty. \)