1. To find \( \int x^2 \sqrt{16 - x^2} \, dx \) by trig substitution, the first step you should make is to let:

(a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)  
(b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)  
(c) \( x = 4 \tan \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)  
(e) \( x = 4 \sec \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)  
(f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(g) none of these

2. To find \( \int \frac{x^3}{\sqrt{x^2 - 16}} \, dx \) by trig substitution, the first step you should make is to let:

(a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)  
(b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)  
(c) \( x = \sin \theta \) and \( dx = \cos \theta \, d\theta \)  
(d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)  
(e) \( x = \sec \theta \) and \( dx = \sec \theta \tan \theta \, d\theta \)  
(f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(g) none of these

3. To find \( \int \frac{\sqrt{x^2 + 16}}{x^2} \, dx \) by trig substitution, the first step you should make is to let:

(a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)  
(b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)  
(c) \( x = \sin \theta \) and \( dx = \cos \theta \, d\theta \)  
(d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)  
(e) \( x = \sec \theta \) and \( dx = \sec \theta \tan \theta \, d\theta \)  
(f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(g) none of these

4. To find \( \int x^3 \sqrt{9 - x^2} \, dx \) by trig substitution, we let \( x = 3 \sin \theta \) and \( dx = 3 \cos \theta \, d\theta \). After simplifying, the integral becomes:

(a) \( \int 243 \sin^3 \theta \cos^2 \theta \, d\theta \)  
(b) \( \int 9 \sin^3 \theta \cos \theta \, d\theta \)  
(c) \( \int 27 \sin^2 \theta \cos^2 \theta \, d\theta \)  
(d) \( \int 243 \sin^3 \theta \cos \theta \, d\theta \)  
(e) \( \int 81 \sin^3 \theta \sqrt{9 - \sin^2 \theta} \cos \theta \, d\theta \)  
(f) \( \int 27 \sin^3 \theta \sqrt{9 - 9 \sin^2 \theta} \, d\theta \)  
(g) none of these

5. To find \( \int x^2 \sqrt{9 + x^2} \, dx \) by trig substitution, we let \( x = 3 \tan \theta \) and \( dx = 3 \sec^2 \theta \, d\theta \). After simplifying, the integral becomes:

(a) \( \int 243 \tan^2 \theta \sec^2 \theta \, d\theta \)  
(b) \( \int 27 \tan^3 \theta \sec \theta \, d\theta \)  
(c) \( \int 81 \tan^2 \theta \sec^3 \theta \, d\theta \)  
(d) \( \int 81 \sec^3 \theta \tan^3 \theta \, d\theta \)  
(e) \( \int 81 \tan^3 \theta \sqrt{9 + \tan^2 \theta} \sec^2 \theta \, d\theta \)  
(f) \( \int 27 \tan^3 \theta \sqrt{9 + 9 \tan^2 \theta} \, d\theta \)  
(g) none of these

6. To find \( \int \cos^6 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:

(a) \( \cos \theta = \cos \theta \) and \( \sin \theta = 1 - \cos^2 \theta \)  
(b) \( \sin \theta = \sin \theta \) and \( \cos \theta = 1 - \sin^2 \theta \)  
(c) \( \cos \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin \theta = \frac{1 - \cos(2\theta)}{2} \)  
(d) \( \cos \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin \theta = 1 - \cos^2 \theta \)  
(e) \( \sin \theta = \frac{1 - \cos(2\theta)}{2} \) and \( \cos \theta = 1 - \sin^2 \theta \)  
(f) \( \sin \theta = 1 - \cos^2 \theta \) and \( \cos \theta = 1 - \sin^2 \theta \)  
(g) \( \cos \theta = \cos \theta \) and \( \sin \theta = 1 - \sin^2 \theta \)  
(h) \( \sin \theta = \sin \theta \) and \( \sin \theta = 1 - \cos^2 \theta \)

7. To find \( \int \cos^7 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:

(a) \( \cos \theta = \cos \theta \) and \( \sin \theta = 1 - \cos^2 \theta \)  
(b) \( \sin \theta = \sin \theta \) and \( \cos \theta = 1 - \sin^2 \theta \)  
(c) \( \cos \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin \theta = \frac{1 - \cos(2\theta)}{2} \)  
(d) \( \cos \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin \theta = 1 - \cos^2 \theta \)  
(e) \( \sin \theta = \frac{1 - \cos(2\theta)}{2} \) and \( \cos \theta = 1 - \sin^2 \theta \)  
(f) \( \sin \theta = 1 - \cos^2 \theta \) and \( \cos \theta = 1 - \sin^2 \theta \)  
(g) \( \cos \theta = \cos \theta \) and \( \sin \theta = 1 - \sin^2 \theta \)  
(h) \( \sin \theta = \sin \theta \) and \( \sin \theta = 1 - \cos^2 \theta \)
8. To find $\int \sec^4 \theta \tan^4 \theta \, d\theta$ you should transform this integral into:

(a) $\int \sec^2 \theta \tan^2 \theta \sec^2 \theta \, d\theta = \int \sec^2 \theta (\sec^2 \theta - 1)^2 \sec^2 \theta \, d\theta = \int \sec^4 \theta - 2 \sec^6 \theta + \sec^4 \theta \, d\theta$

(b) $\int \sec^2 \theta \tan^4 \theta \sec^2 \theta \, d\theta = \int (1 + \tan^2 \theta) \tan^4 \theta \sec^2 \theta \, d\theta = \int (\tan^4 \theta + \tan^6 \theta) \sec^2 \theta \, d\theta$

(c) $\int \sec^3 \theta \tan^3 \theta \sec \theta \tan \theta \, d\theta = \int \sec^2 \theta \sec \theta \tan^2 \theta \sec \theta \tan \theta \, d\theta = \int \tan^2 \theta (\sec \theta \tan \theta)^2 \sec^2 \theta \, d\theta$

(d) $\int \sec^2 \theta \tan^4 \sec^2 \theta \, d\theta = \int (1 + \tan^2 \theta)(\sec^2 \theta - 1) \sec^2 \theta \, d\theta$

(e) $\int \sec^2 \theta \tan^2 \sec^2 \theta \, d\theta = \int (1 + \tan^2 \theta)(\sec^2 \theta - 1) \sec \theta \tan \theta \, d\theta$

(f) $\int (1 + \tan^2 \theta)^2 (\sec^2 \theta - 1)^2 ; \, d\theta$

9. Find the correct standard form of the partial fractions decomposition for $\frac{x^3 + x + 200}{(x^2 + x - 2)(x - 1)^2(x^2 + 4)^2}$. Do not find the constants.

(a) $\frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2}$

(b) $\frac{A}{x^2 + x - 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2}$

(c) $\frac{Ax + B}{x^2 + x - 2} + \frac{C}{x - 1} + \frac{D}{(x - 1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2}$

(d) $\frac{A}{x^2 + x - 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{Dx}{x^2 + 4} + \frac{Ex}{(x^2 + 4)^2}$

(e) $\frac{A}{x^2 + x - 2} + \frac{B}{x - 1} + \frac{Cx + D}{(x - 1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{Gx}{(x^2 + 4)^2}$

(f) $\frac{A}{x^2 + x - 2} + \frac{B}{x - 1} + \frac{Cx + D}{(x - 1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{G}{x^2 + 4} + \frac{H}{(x^2 + 4)^2}$

10. In the partial fraction expansion $\frac{x}{(x - 1)(x - 4)^2(x + 2)} = \frac{A}{x - 1} + \frac{B}{(x - 4)^2} + \frac{C}{x - 4} + \frac{D}{x + 2}$, the value of $A$ is

(a) $1/81$  
(b) $1/3$  
(c) $-1/3$  
(d) $1/9$  
(e) $2/9$  
(f) none of these

11. Given that the partial fractions expansion of $\frac{x^3}{(x - 1)^2(x^2 + 1)}$ is $\frac{\frac{1}{2}}{(x - 1)^2} + \frac{\frac{1}{2}}{x - 1} - \frac{\frac{1}{2}}{x^2 + 1}$, find $\int \frac{x^3}{(x - 1)^2(x^2 + 1)} \, dx$

12. Evaluate $\int x^2 + 1 \, dx$ using substitution with $u = \frac{x}{x^2 + 1}$ and $du = \frac{x^2}{x^2 + 1} \, dx$.

13. (A) The partial fractions expansion of $\frac{x + 7}{x^2 - x - 2}$ is ________________

(B) Evaluate $\int \frac{x + 7}{x^2 - x - 2} \, dx$ using partial fractions.

14. To find $\int x^2 + 10 \, dx$, the appropriate method you should use is:

(A) substitution with $u = \frac{x}{x^2 + 1}$ and $du = \frac{x}{x^2 + 1} \, dx$

(B) parts with $u = \frac{x}{x^2 + 1}$ and $dv = \frac{x}{x^2 + 1} \, dx$

(C) trig substitution with $x = \frac{\sin \theta}{\cos \theta}$ and $dx = \frac{-\sin^2 \theta}{\cos \theta} \, d\theta$

(D) None of the three methods. It can be done directly.

Evaluate the integral and show your work.
15. To find \[ \int \frac{y+1}{\sqrt{y}} \, dy, \] the appropriate method you should use is:

(A) substitution with \( u = \ldots \) and \( du = \ldots \)

(B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)

(C) trig substitution with \( x = \ldots \) and \( dx = \ldots \)

(D) None of the three methods. It can be done directly.

**Evaluate the integral and show your work**

16. To find \[ \int x^5 \sqrt{x^2 + 1} \, dx, \] the appropriate method you should use is:

(A) substitution with \( u = \ldots \) and \( du = \ldots \)

(B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)

(C) trig substitution with \( x = \ldots \) and \( dx = \ldots \)

(D) None of the three methods. It can be done directly.

**Do not evaluate the integral**

17. To find \[ \int_0^2 \frac{t^3}{t^4 + 11} \, dt \] the appropriate method you should use is:

(A) substitution with \( u = \ldots \) and \( du = \ldots \)

(B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)

(C) trig substitution with \( t = \ldots \) and \( dt = \ldots \)

(D) None of the three methods. It can be done directly.

**Evaluate the integral and show your work**

18. To find \[ \int s^4 + 11 / s^3 \, ds \] the appropriate method you should use is:

(A) substitution with \( u = \ldots \) and \( du = \ldots \)

(B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)

(C) trig substitution with \( s = \ldots \) and \( ds = \ldots \)

(D) None of the three methods. It can be done directly.

**Evaluate the integral and show your work**

19. To find \[ \int \frac{x^5}{\sqrt{x^2 - 1}} \, dx, \] the appropriate method you should use is:

(A) substitution with \( u = \ldots \) and \( du = \ldots \)

(B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)

(C) trig substitution with \( x = \ldots \) and \( dx = \ldots \)

(D) None of the three methods. It can be done directly.

**Do not evaluate the integral.**

20. To find \[ \int \ln z \, dz \] the appropriate method you should use is:

(A) substitution with \( u = \ldots \) and \( du = \ldots \)

(B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)

(C) trig substitution with \( z = \ldots \) and \( dz = \ldots \)

(D) None of the three methods. It can be done directly.

**Evaluate the integral and show your work**
21. To find $\int_0^2 xe^x \, dx$ the appropriate method you should use is:
   (A) substitution with $u = \underline{\quad}$ and $du = \underline{\quad}$
   (B) parts with $u = \underline{\quad}$, $dv = \underline{\quad}$, $du = \underline{\quad}$ and $v = \underline{\quad}$
   (C) trig substitution with $x = \underline{\quad}$ and $dx = \underline{\quad}$
   (D) None of the three methods. It can be done directly.

   **Evaluate the integral and show your work**

22. To find $\int re^r \, dr$ the appropriate method you should use is:
   (A) substitution with $u = \underline{\quad}$ and $du = \underline{\quad}$
   (B) parts with $u = \underline{\quad}$, $dv = \underline{\quad}$, $du = \underline{\quad}$ and $v = \underline{\quad}$
   (C) trig substitution with $r = \underline{\quad}$ and $dr = \underline{\quad}$
   (D) None of the three methods. It can be done directly.

   **Evaluate the integral and show your work**

23. To find $\int \theta^3 \cos \theta \, d\theta$ the appropriate method you should use is:
   (A) substitution with $u = \underline{\quad}$ and $du = \underline{\quad}$
   (B) parts with $u = \underline{\quad}$, $dv = \underline{\quad}$, $du = \underline{\quad}$ and $v = \underline{\quad}$
   (C) trig substitution with $\theta = \underline{\quad}$ and $d\theta = \underline{\quad}$
   (D) None of the three methods. It can be done directly.

   **Do not evaluate the integral.**

24. Explain why $\int_3^5 \frac{1}{\sqrt{4x-12}} \, dx$ is an “improper” integral. Evaluate it if it converges. If it diverges, show why.

25. Evaluate $\int_5^\infty \frac{1}{2x+1} \, dx$ if it converges. If it diverges, show why.

26. Evaluate $\int_4^\infty \frac{1}{(2x+1)^{3/2}} \, dx$ if it converges. If it diverges, show why.

27. Use (a) Simpson’s rule, (b) the trapezoidal rule, (c) a left end point Riemann sum, and (d) a right end point Riemann sum all with 6 subintervals to estimate $\int_1^4 f(x) \, dx$ if you know the function values from the table.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2.5</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>3.5</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

28. Use (a) Simpson’s rule and (b) the trapezoidal rule both with 4 subintervals to estimate $\int_1^4 \frac{1}{x} \, dx$.

29. Problem 23 Sec 7.6 (page 485).

30. Example 5 Sec 7.6 (page 484).
31. The distances across a lake are 8, 70, 60, 45, 10, and 5 meters as shown. They were measured at 6 meter intervals. Use (I) the Trapezoidal Rule and (II) Simpson’s rule to estimate the area of the lake. Please show your work.

(I) The Trapezoidal Rule gives:
(a) 1164 (b) 388 (c) 594 (d) 990 (e) 1140 (f) 995

(II) Simpson’s Rule gives:
(a) 628 (b) 1980 (c) 594 (d) 990 (e) 1140 (f) 1256

32. Let $g$ be a function on $[1, 4]$ such that $-6 \leq g^{(4)}(x) \leq 3$ for $1 \leq x \leq 4$.

(I) In the error formula for Simpson’s Rule, what is the best value of $K$?
(a) 0 (b) 4 (c) $-6$ (d) 6 (e) 5 (f) 1 (g) none of these.

(II) What is the smallest number of subintervals one should use in order to assure that Simpson’s Rule approximates $\int_{1}^{4} g(x) \, dx$ to within $10^{-4}$?
(a) 25 (b) 13 (c) 8100 (d) 16 (e) 17 (f) 18 (g) none of these.

33. Let $f$ be a function on $[1, 4]$ such that $-1 \leq f''(x) \leq 5$ for all $x$ in $[1, 4]$. What is the smallest number of subintervals one should use in order to assure that the Trapezoidal Rule approximates $\int_{1}^{4} f(x) \, dx$ to within $10^{-4}$?
(a) 1125 (b) 336 (c) 112 (d) 11,250 (e) 335 (f) 1060 (g) none of these.