1. Exactly two of the following are true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$. Mark only those two.

(a) It converges absolutely by the ratio test.
(b) It converges by the ratio test.
(c) It diverges by the ratio test.
(d) It converges absolutely because a $p$–series with $p < 1$ converges.
(e) It converges conditionally.
(f) It converges because it is absolutely convergent and an absolutely convergent series is also convergent.
(g) It converges by the Alternating Series test.
(h) It diverges by the Alternating Series test.
(i) It diverges because it’s a $p$–series with $p < 1$.
(j) It converges by the limit comparison test.
(k) It diverges by the comparison test.

2. Exactly three of the following are true about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^3}}$. Mark only those three.

(a) It converges absolutely by the ratio test.
(b) It converges by the ratio test.
(c) It diverges by the ratio test.
(d) It converges absolutely because a $p$–series with $p > 1$ converges.
(e) It converges conditionally.
(f) It converges because it is absolutely convergent and an absolutely convergent series is also convergent.
(g) It converges by the Alternating Series test.
(h) It diverges by the Alternating Series test.
(i) It diverges because it’s a $p$–series with $p > 1$.
(j) It diverges by the limit comparison test.

3. Exactly two of the following are true about the series $\sum (-1)^n \frac{2n^2}{3 + 5n^2}$. Mark only those two.

(a) It converges absolutely by the ratio test.
(b) It converges by the ratio test.
(c) It diverges by the ratio test.
(d) It converges absolutely because a $p$–series with $p > 1$ converges.
(e) It converges conditionally.
(f) It converges because it is absolutely convergent and an absolutely convergent series is also convergent.
(g) It converges by the Alternating Series test.
(h) It diverges by the Alternating Series test.
(i) It diverges because it’s a $p$–series with $p > 1$.
(j) It diverges by the $n^{th}$ term divergence test.
4. Exactly two of the following are true about the series \( \sum_{n=1}^{\infty} \frac{n}{3^n} \). Mark only those two.

(a) It converges absolutely by the ratio test.
(b) It converges by the ratio test.
(c) It diverges by the ratio test.
(d) It converges conditionally.
(e) It converges by the Alternating Series test.
(f) It diverges because it’s a \( p \)-series with \( p > 1 \).
(g) It diverges by the comparison test.
(h) It diverges because it is geometric with \( r = 3 \).
(i) It converges because it is geometric with \( r = \frac{1}{3} \).

5. Exactly one of the following is true about the series \( \sum_{n=1}^{\infty} (-1)^n \frac{n^4}{2^n} \). Mark only that one.

(a) It converges absolutely by the ratio test.
(b) It converges by the ratio test.
(c) It diverges by the ratio test.
(d) The ratio test is inconclusive for this series.
(e) It converges conditionally.
(f) It converges by the Alternating Series test.
(g) It diverges because it’s a \( p \)-series with \( p > 1 \).
(h) It diverges because it is geometric with \( r > 1 \).
(i) It converges because it is geometric with \( r < 1 \).

6. Exactly one of the following is true about the series \( \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3} \). Mark only that one.

(a) It converges by the ratio test.
(b) It converges by the ratio test.
(c) It converges by the Alternating Series test.
(d) It converges by the comparison test because \( 0 \leq \frac{\sin^2 n}{n^3} \leq \frac{1}{n^3} \).
(e) It diverges by the comparison test because \( \frac{\sin^2 n}{n^3} \geq \frac{1}{n^3} \).

7. If you apply the limit comparison test to \( \sum \frac{2\sqrt{n} + 7}{3n + 5} \), the series you should use for comparison is

(a) \( \sum \frac{1}{n^2} \)  
(b) \( \sum \frac{1}{n^2} \)  
(c) \( \sum n^\frac{1}{2} \)  
(d) \( \sum \frac{1}{n^2} \)  
(e) \( \sum \frac{1}{3n} \)  
(g) none of these

8. Apply the limit comparison test to \( \sum \frac{2\sqrt{n} + 7}{3n + 5} \) to decide whether it converges or diverges.

9. Apply the integral test to determine if the series \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} \) converges or diverges.

10. Apply the integral test to determine if the series \( \sum_{n=3}^{\infty} \frac{1}{n \ln n} \) converges or diverges.
11. Apply the Ratio Test to determine if the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}3^n}{n!} \) converges absolutely, (b) converges conditionally, or (c) diverges.

12. Apply the Ratio Test to determine if the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^n n^2}{5^{n+1}} \) converges absolutely, (b) converges conditionally, or (c) diverges.

13. What is the solution to the inequality \(|2x - 1| < 3|\)?
   (a) \(1 < x < 2\) (b) \(-1 < x < 2\) (c) \(-2 < x < 2\) (d) \(-1 < x < 1\) (e) \(0 < x < 2\) (f) \(-2 < x < 0\) (g) none of these

14. Apply the ratio test to find the largest open interval on which series \( \sum_{n=0}^{\infty} \frac{(-1)^n}{8^{n+1}}(x - 2)^{3n} \) converges absolutely. Do not check the endpoints.
   (a) \((-2, 0)\) (b) \((-\frac{1}{2}, \frac{1}{2})\) (c) \((-8, 8)\) (d) \((-1, 2)\)
   (e) \((-2, 2)\) (f) \((-\frac{1}{8}, \frac{1}{8})\) (g) \((0, 4)\) (h) none of these.

15. If the interval of convergence of \( \sum a_n(x - x_0)^n \) is \([-5, 7]\) what is its radius of convergence?
   (a) \(\frac{1}{2}\) (b) 4 (c) 5 (d) 7 (e) 12 (f) 6 (g) 1 (h) none of these.

16. If \(|x| < 1\), \( \sum_{n=0}^{\infty} x^n = \)
   (a) \(e^x\) (b) \(\frac{x}{1 - x}\) (c) \(\cos x\) (d) \(\ln x\) (e) \(\frac{1}{1 - x}\) (f) None of these.

17. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n = \)
   (a) \(e^x\) (b) \(\sin x\) (c) \(\cos x\) (d) \(\ln x\) (e) \(\frac{1}{1 - x}\) (f) None of these.

18. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} n^{2n} = \)
   (a) \(0\) (b) 1 (c) \(-1\) (d) \(e\) (e) \(\frac{1}{e}\) (f) \(\sqrt{e}\) (g) \(e^2\) (f) None of these.

19. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} n^{2n+1} = \)
   (a) \(0\) (b) 1 (c) \(-1\) (d) \(e\) (e) \(\frac{1}{e}\) (f) \(\sqrt{e}\) (g) \(e^2\) (f) None of these.

20. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \)
   (a) \(0\) (b) 1 (c) \(-1\) (d) \(e\) (e) \(\frac{1}{e}\) (f) \(\sqrt{e}\) (g) \(e^2\) (f) None of these.
21. \( \sum_{n=0}^{\infty} \frac{1}{2^n n!} = \)
(a) 0  (b) 1  (c) -1  (d) e  (e) \( \frac{1}{e} \)  (f) \( \sqrt{e} \)  (g) \( e^2 \) (f) None of these.

22. \( \sum_{n=0}^{\infty} \frac{2^n}{n!} = \)
(a) 0  (b) 1  (c) -1  (d) e  (e) \( \frac{1}{e} \)  (f) \( \sqrt{e} \)  (g) \( e^2 \) (f) None of these.

23. Evaluate the following argument by circling whether each line is correct or not.
(a) \( \sqrt{n} + n \leq n^2 + n^2 = 2n^2 \). (correct or not?)
(b) Therefore \( \frac{1}{\sqrt{n} + n} \geq \frac{1}{2n^2} \). (correct or not?)
(c) But \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) is convergent because it is one-half times a convergent \( p \)-series \( (p = 2) \). (correct or not?)
(d) Lines (a) - (c) show that the series \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n} \) converges by the comparison test. (correct or not?)

24. Evaluate the following argument by circling whether each line is correct or not.
(a) \( \sqrt{n} + n \leq n + n = 2n \). (correct or not?)
(b) Therefore \( \frac{1}{\sqrt{n} + n} \geq \frac{1}{2n} \). (correct or not?)
(c) But \( \sum_{n=1}^{\infty} \frac{1}{n} \) is divergent because it is one-half times the divergent harmonic series. (correct or not?)
(d) Lines (a) - (c) show that the series \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n} \) diverges by the comparison test. (correct or not?)

25. Evaluate the following argument by circling whether each line is correct or not.
(a) If you divide \( \frac{1}{\sqrt{n} + n} \) by \( \frac{1}{n^2} \) you get \( \frac{n^2}{\sqrt{n} + n^2} \). (correct or not?)
(b) \( \lim_{n \to \infty} \frac{n^2}{\sqrt{n} + n^2} = 1 \). (correct or not?)
(c) But \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) is convergent because it is a \( p \)-series \( (p > 1) \). (correct or not?)
(d) Lines (a) - (c) show that the series \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n^2} \) converges by the limit comparison test. (correct or not?)

26. \( \sum_{n=0}^{\infty} (-1)^n \frac{2^{n-1}}{7n+2} \) equals which of these?
(a) \(- \frac{1}{441} \)  (b) \(- \frac{1}{245} \)  (c) \(- \frac{2}{441} \)  (d) \( \frac{1}{126} \)  (e) \( \frac{7}{18} \)  (f) \( \frac{1}{70} \)  (g) None of these - it diverges.

27. \( \sum_{n=0}^{\infty} (-1)^n \frac{4^{n-1}}{3n+2} \) equals which of these?
(a) \(- \frac{1}{12} \)  (b) \( \frac{1}{84} \)  (c) \( \frac{1}{9} \)  (d) \( \frac{1}{63} \)  (e) \( \frac{4}{3} \)  (f) \(- \frac{4}{3} \)  (g) None of these - it diverges
28. Suppose that $\sum_{n=1}^{\infty} \frac{n}{3^n} = \frac{3}{4}$. What is $\sum_{n=3}^{\infty} \frac{n}{3^n}$?

(a) $\frac{5}{36}$  (b) $\frac{9}{34}$  (c) $\frac{2}{3}$  (d) $\frac{1}{12}$  (e) $\frac{7}{34}$  (f) $\frac{7}{36}$  (g) None of these.

29. Which of the following is the Maclaurin series for $\cos(x^2)$?

(a) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^{4n}}{(2n+1)!}$
(b) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+4}}{2n+1}$
(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n}$
(d) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n)!} x^{4n+2}$
(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n+2}$
(f) None of these.

30. Which of the following is the Maclaurin series for $xe^x$?

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{n+1}$
(b) $\sum_{n=0}^{\infty} \frac{1}{n!} x^{n+1}$
(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{n-1}$
(d) $\sum_{n=0}^{\infty} \frac{1}{n^n!} x^{n+1}$
(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$
(f) $\sum_{n=0}^{\infty} \frac{x}{n!} x^{n+1}$
(g) $\sum_{n=0}^{\infty} \frac{(-1)^n x}{n!} x^n$
(h) None of these.

31. Which of the following is the Maclaurin series for $x \sin(x^2)$.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+2}$
(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+3}$
(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+3}$
(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+2)!} x^{4n+2}$
(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n+2}$
(f) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+3}$
(g) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n+2}$
(h) None of these.

32. Show how the Maclaurin series for $e^x$ is used to determine that $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2}$.

33. Show how the Maclaurin series for $\cos x$ is used to determine that $\lim_{x \to 0} \frac{\cos x - x^2 + \frac{x^4}{24} + \frac{x^2}{2} - 1}{x^6} = -\frac{1}{720}$.
34. Which of the following is the partial sum of the first 4 terms of the Taylor series for \( f(x) = \sqrt[3]{x} \) about \( a = 8 \)?

(a) \( 2 + \frac{1}{12} (x - 8) - \frac{1}{144} (x - 8)^2 + \frac{5}{10318} (x - 8)^3 \)

(b) \( 2 + \frac{1}{12} (x - 8) - \frac{1}{288} (x - 8)^2 + \frac{5}{20736} (x - 8)^3 \)

(c) \( \frac{1}{2} - 6(x - 8) - \frac{1}{8} (x - 8)^2 + \frac{4}{81} (x - 8)^3 \)

(d) \( 2 + \frac{1}{12} (x - 8) - \frac{1}{288} (x - 8)^2 + \frac{15}{10363} (x - 8)^3 \)

(e) \( 2 + \frac{1}{12} x - \frac{1}{288} x^2 + \frac{5}{20736} x^3 \)

(f) \( 2 + \frac{1}{12} x - \frac{1}{144} x^2 + \frac{5}{10318} x^3 \)

(g) \( \frac{1}{2} - 6x - \frac{1}{8} x^2 + \frac{4}{81} x^3 \)

(h) \( 2 + \frac{1}{12} (x) - \frac{1}{288} x^2 + \frac{15}{10363} x^3 \)

35. Which of the following is the partial sum of the first 4 terms of the Taylor series for \( f(x) = \frac{1}{\sqrt[3]{x}} \) about \( a = 1 \)?

(a) \( 1 - \frac{2}{3} (x - 1) + \frac{5}{9} (x - 1)^2 - \frac{40}{81} (x - 1)^3 \)

(b) \( 1 - \frac{1}{3} (x - 1) + \frac{2}{9} (x - 1)^2 - \frac{14}{81} (x - 1)^3 \)

(c) \( 1 + \frac{2}{3} (x - 1) - \frac{1}{9} (x - 1)^2 + \frac{4}{81} (x - 1)^3 \)

(d) \( 1 + \frac{1}{3} (x - 1) - \frac{1}{9} (x - 1)^2 + \frac{5}{81} (x - 1)^3 \)

(e) \( 1 - \frac{1}{3} (x - 1) + \frac{8}{9} (x - 1)^2 - \frac{28}{27} (x - 1)^3 \)

(f) \( 1 + \frac{1}{3} x - \frac{1}{9} x^2 + \frac{5}{81} x^3 \)

(g) \( 1 - \frac{1}{3} x + \frac{2}{9} x^2 - \frac{14}{81} x^3 \)

(h) \( 1 + \frac{2}{3} x - \frac{1}{9} x^2 + \frac{4}{81} x^3 \)
36. Which of the following is the partial sum of the first 4 terms of the Maclaurin series for $f(x) = \sqrt{x+1}$.

(a) $1 + \frac{3}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{5}{128}(x-1)^3$

(b) $1 + \frac{1}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{7}{128}(x-1)^3$

(c) $1 - \frac{3}{4}(x-1) + \frac{21}{32}(x-1)^2 - \frac{77}{128}(x-1)^3$

(d) $1 - \frac{1}{4}(x-1) + \frac{5}{32}(x-1)^2 - \frac{15}{128}(x-1)^3$

(e) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

(f) $1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3$

(g) $1 + \frac{1}{4}x - \frac{3}{16}x^2 + \frac{21}{64}x^3$

(h) $1 - \frac{1}{4}x + \frac{5}{32}x^2 - \frac{15}{128}x^3$

37. Which of the following is the partial sum of the first 4 terms of the Maclaurin series for $f(x) = \frac{1}{\sqrt{x+1}}$?

(a) $1 + \frac{3}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{5}{128}(x-1)^3$

(b) $1 + \frac{1}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{7}{128}(x-1)^3$

(c) $1 - \frac{3}{4}(x-1) + \frac{21}{32}(x-1)^2 - \frac{77}{128}(x-1)^3$

(d) $1 - \frac{1}{4}(x-1) + \frac{5}{32}(x-1)^2 - \frac{15}{128}(x-1)^3$

(e) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

(f) $1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3$

(g) $1 - \frac{1}{4}x + \frac{5}{16}x^2 - \frac{45}{64}x^3$

(h) $1 - \frac{1}{4}x + \frac{5}{32}x^2 - \frac{15}{128}x^3$

In the following, circle “T” if it is always true and “F” otherwise.

1. **T** F $\lim_{n \to \infty} \frac{6n+9}{3n+2} = 2$.

2. **T** F $\lim_{n \to \infty} \frac{n}{5n+1} = \frac{1}{5}$, therefore $\sum_{n=1}^{\infty} \frac{n}{5n+1}$ diverges.

3. **T** F $\sum_{n=0}^{\infty} \frac{(-1)^n3^{n+1}}{4^n} = 12$.

4. **T** F If $0 \leq a_n \leq \frac{1}{n^2}$ for every $n$, then $\sum a_n$ converges.
5. T F If $0 \leq a_n \leq \frac{1}{\sqrt{n}}$ for every $n$, then $\sum a_n$ diverges.

6. T F The Limit Comparison Test says that if $a_n \geq 0$, $b_n \geq 0$, $\lim_{n \to \infty} \frac{a_n}{b_n} = c > 0$, and $\sum a_n$ diverges then $\sum b_n$ diverges too.

7. T F If $\frac{n}{n+1} \leq a_n \leq \frac{n^2}{n^2 + 1}$ for every $n$, then $a_n \to 1$.

8. T F $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$ converges.

9. T F $\lim_{n \to \infty} \frac{n}{n+1} = 1$, which is not zero, therefore $\frac{n}{n+1}$ diverges by the $n^{th}$ term divergence test.

10. T F $\lim_{n \to \infty} \frac{2n}{n+1} = 2$, therefore $\sum_{n=1}^{\infty} \frac{2n}{n+1}$ diverges because the $n^{th}$ term does not go to 0.

11. T F If $\sum_{n=1}^{\infty} c_n = 1$ then it diverges by the $n^{th}$ term divergence test.

12. T F If $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$, then $c_n = \frac{f^{(n)}(x)}{n!}$.

13. T F If $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$, then $c_n = f^{(n)}(a)$.

14. T F If $\{b_n\}$ is increasing and bounded then it must converge.

15. T F $\left\{ \frac{1}{n} \right\}$ is a decreasing sequence.

16. T F $\left\{ (-1)^n \right\}$ is a monotonic sequence.

17. T F $\left\{ \frac{n}{2} \right\}$ is a monotonic sequence.

18. T F If $\lim_{n \to \infty} a_n \neq 0$ then it diverges.

19. T F If $\lim_{n \to \infty} a_n = 0$ then $\lim_{n \to \infty} |a_n| = 0$

20. T F If $\lim_{n \to \infty} |a_n| = 0$ then $\lim_{n \to \infty} a_n = 0$

21. T F If $\lim_{n \to \infty} a_n = 0$ then $\{a_n\}$ converges.

22. T F If $\lim_{n \to \infty} a_n = 0$ then $\sum a_n$ converges.

23. T F If $\sum a_n$ converges then $\lim_{n \to \infty} a_n = 0$.

24. T F If $\sum a_n$ converges, then $\sum |a_n|$ converges.

25. T F If $\sum |a_n|$ converges, then $\sum a_n$ converges.
26. T F $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

27. T F $\left\{ \frac{1}{n} \right\}$ diverges because it is the harmonic series.

28. T F $\sum_{n=1}^{\infty} \frac{1}{n}$ is decreasing.

29. T F If the sequence $\{a_n\}$ converges to 11, so does $\{a_{n+2}\}$.

30. T F $\sum_{n=1}^{\infty} r^n$ converges to $\frac{r}{1-r}$.

31. T F $\sum_{n=0}^{\infty} r^n$ converges to $\frac{1}{1-r}$ if $-1 < r < 1$.

32. T F $\sum_{n=0}^{\infty} a_n$ is the same as $\sum_{n=1}^{\infty} a_{n-1}$.

33. T F $\sum_{n=0}^{\infty} \frac{5^n}{3^n}$ diverges.

34. T F $\sum_{n=0}^{\infty} 1.0001^n$ diverges.

35. T F $\sum_{n=0}^{\infty} 0.999^n$ diverges.

36. T F $\sum_{n=1}^{\infty} n^n$ is a geometric series.

37. T F If $\sum_{n=1}^{k} |a_n| \leq 201$ for every positive integer $k$, then $\sum_{n=1}^{\infty} a_n$ converges.

38. T F $\sum_{n=1}^{\infty} \frac{1}{n}$ converges to 0.

39. T F $\sum_{n=1}^{\infty} 3^n = \frac{3}{1-3} = \frac{-3}{2}$.

40. T F $\sum_{n=1}^{\infty} r^{n+2} = \frac{r^3}{1-r}$ for $|r| < 1$.

41. T F According to the “squeeze theorem”, if $\{b_n\}$ converges to 3 and $\{a_n\}$ converges to 3 and $b_n \leq c_n \leq a_n$ then $\{c_n\}$ converges to 3.

42. T F The “squeeze theorem” shows that $\left\{ \frac{\cos n}{n} \right\}$ converges to 0.