1. Use Euler’s method with step size 0.1 to estimate \( y(1.3) \) if \( \frac{dy}{dx} = x + 2y \) and \( y(1) = 3 \). Make a table showing the steps leading to the answer. (Recall \( y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}) \))

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>3.0</td>
<td>7.0</td>
</tr>
<tr>
<td>1.1</td>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>1.2</td>
<td>4.55</td>
<td>8.5</td>
</tr>
<tr>
<td>1.3</td>
<td>5.58</td>
<td>10.3</td>
</tr>
</tbody>
</table>

The final answer is \( y(1.3) = 5.58 \)

2. A vat with 2000 liters of water has 200 kg of salt in solution. A solution containing 0.05 kg of salt per liter is running into the tank at 10 liters per minute while it is draining at the same rate.

(I) Which equation models the amount of salt in the tank at time \( t \)? Answer: (a)

(a) \( y' = 0.5 - y/200 \)  
(b) \( y' = 0.5 - y/2000 \)  
(c) \( y' = 0.05 - y/200 \)  
(d) \( y' = 20 - y/100 \)  
(e) \( y' = 5 - y/2000 \)  
(f) \( y' = 0.005 - y/200 \)

(II) Find a formula for the amount of salt in the tank at time \( t \). Answer: \( y = 100 + 100e^{-t/200} \).

3. Assume a population obeys the logistic equation \( \frac{dP}{dt} = 21P - 0.0042P^2 \). What is the carrying capacity? If the initial population is \( P(0) = 6000 \), would the population decrease or increase? Answer: 5000, decreasing.

4. Only one of these graphs can be the solution to \( y' = e^{-y} \). Which one is it? Answer: (d). (Reasons: \( y \) must be increasing and as \( y \to \infty \), \( y' \to 0 \).)

5. Sketch the slope field for \( y' = 3x + y \) at \((-1, 1)\); \((0.5, -1.5)\) and \((0, 1)\). Note that at \((-1, 1)\) \( y' = -1 \); at \((0.5, -1.5)\) \( y' = 0 \); at \((0, 1)\) \( y' = 1 \).
6. Using the given slope field, sketch the graph that satisfies the initial condition \( y(0) = 0 \).

7. Which differential equations below are separable? \( \text{Answer: (a) (c) (d) (e).} \)
   (a) \( \frac{dy}{dx} = xy \) (b) \( \frac{dy}{dx} = x + y \) (c) \( \frac{dy}{dx} = \frac{\ln y}{x} \) (d) \( \frac{dy}{dx} = xy^2 + x \cos(y) \) (e) \( \frac{dy}{dx} = \frac{y}{e^x} \) (f) \( \frac{dy}{dx} = \sin(yx) \)

8. Find the general solution to the differential equation \( y' = 2xy^2 \). \( \text{Answer: (a)} \)
   (a) \( y = -\frac{1}{x^2 + C} \) (b) \( y = e^{x^2 + C} \) (c) \( y = e^{\frac{x^2}{2} + C} \) (d) \( y = Ce^{x^2} \) (e) \( y = \frac{C}{x^2} \) (f) \( y = \frac{1}{x^2 - C} \) (g) none of these

9. Find the general solution to the differential equation \( \frac{dy}{dt} = 2ty + t \). \( \text{Answer: (f)} \)
   (a) \( y = Ce^{t^2} \) (b) \( y = 2e^{t^2} - \frac{1}{2} \) (c) \( y = \frac{e^{t^2} - C}{2} \) (d) \( y = Ce^{t^2} - \frac{1}{2} \) (e) \( y = Ce^{t^2} - \frac{C}{2} \) (f) \( y = Ce^{t^2} - \frac{1}{2} \) (g) none of these

10. Solve \( y' = x^2y, y(0) = 3 \). \( \text{Answer: (c)} \)
    (a) \( y = 3e^{x^2} \) (b) \( y = e^{\frac{x^3}{3}} \) (c) \( y = 3e^{\frac{x^3}{3}} \) (d) \( y = 3 + \frac{x^3}{3} \) (e) \( y = e^{3x^3} \) (f) \( y = 3e^{3x^3} \) (g) none of these

11. Solve \( \frac{dy}{dt} = y^2 \sin t, y(0) = 3 \). \( \text{Answer: (d)} \)
    (a) \( y = \frac{1}{\sin t + \frac{1}{3}} \) (b) \( y = \frac{\sin t}{\sqrt{\cos t - \frac{3}{4}}} \) (c) \( y = 3e^{\cos t - 1} \) (d) \( y = \frac{1}{\cos t - \frac{3}{3}} \) (e) \( y = e^{\frac{3}{2} - \cos t} \) (f) \( y = \frac{3}{\cos t + 1} \) (g) none of these

12. A mass of 2 grams is located at \((3, 2)\), a mass of 3 grams is located at \((-1, 4)\), and a mass of 7 grams is located at \((2, 3)\).
   (I) What are moments of this system about the \( x \)-axis and the \( y \)-axis? \( \text{Answer: (e)} \)
   (a) \( M_x = 23 \) \( M_y = 33 \) (b) \( M_x = 17 \) \( M_y = 33 \) (c) \( M_x = 17 \) \( M_y = 37 \)
   (d) \( M_x = 37 \) \( M_y = 23 \) (e) \( M_x = 37 \) \( M_y = 17 \) (f) \( M_x = 5 \) \( M_y = 7 \)
   (II) What is the centroid of this system? \( \text{Answer: (c)} \)
   (a) \( \bar{x} = \frac{33}{12}, \bar{y} = \frac{23}{12} \) (b) \( \bar{x} = \frac{37}{12}, \bar{y} = \frac{17}{12} \) (c) \( \bar{x} = \frac{17}{12}, \bar{y} = \frac{37}{12} \)
   (d) \( \bar{x} = \frac{37}{12}, \bar{y} = \frac{17}{12} \) (e) \( \bar{x} = \frac{31}{12}, \bar{y} = \frac{23}{12} \) (f) \( \bar{x} = \frac{5}{12}, \bar{y} = \frac{7}{12} \)
13. A region is bounded by \( y = f(x) \) between \( x = a \) and \( x = b \) in the first quadrant. It is known that \( \int_{a}^{b} k \) for \( x \) is \( 20 \), \( \int_{a}^{b} f^2(x) \) for \( dx \) is \( 16 \), and \( \int_{a}^{b} f(x) \) for \( dx \) is \( 2 \).

(I) Which choice below is the moment of the region about the \( x \)-axis? Answer: (h)

(II) Which choice below is the moment of the region about the \( y \)-axis? Answer: (g)

(III) Which choice below is the \( x \)-coordinate of centroid of the region \( (\bar{x}) \)? Answer: (i)

(IV) Which choice below is the \( y \)-coordinate of centroid of the region \( (\bar{y}) \)? Answer: (j)

(a) \( \frac{1}{2} \)  (b) \( \frac{1}{4} \)  (c) \( \frac{1}{10} \)  (d) \( \frac{1}{16} \)  (e) \( 16 \)  (f) \( 2 \)  (g) \( 20 \)  (h) \( 8 \)  (i) \( 10 \)  (j) \( 4 \)

14. Let \( R \) be the region in the first quadrant bounded between \( y = \sqrt[3]{x} \), the \( x \)-axis and the line \( x = 8 \). What are moments of the region \( R \) about the \( x \)-axis and the \( y \)-axis? Answer: (a)

(a) \( M_x = \frac{48}{5} \), \( M_y = \frac{384}{7} \)  
(b) \( M_x = \frac{7}{5} \), \( M_y = \frac{38}{7} \)  
(c) \( M_x = \frac{8}{7} \), \( M_y = \frac{34}{5} \)  
(d) \( M_x = \frac{351}{7} \), \( M_y = \frac{98}{5} \)  
(e) \( M_x = \frac{36}{5} \), \( M_y = \frac{291}{7} \)  
(f) \( M_x = \frac{384}{7} \), \( M_y = \frac{48}{5} \)

15. Let \( R \) be the region in the first quadrant bounded between \( y = x^3 \), the \( x \)-axis and the line \( x = 1 \). What are the coordinates of the centroid of this region \( R \)? Answer: (e)

(a) \( \bar{x} = \frac{5}{7} \), \( \bar{y} = \frac{44}{4} \)  
(b) \( \bar{x} = \frac{4}{5} \), \( \bar{y} = \frac{1}{5} \)  
(c) \( \bar{x} = \frac{1}{7} \), \( \bar{y} = \frac{4}{5} \)  
(d) \( \bar{x} = \frac{4}{14} \), \( \bar{y} = \frac{1}{5} \)  
(e) \( \bar{x} = \frac{4}{5} \), \( \bar{y} = \frac{2}{7} \)  
(f) \( \bar{x} = \frac{1}{2} \), \( \bar{y} = \frac{3}{4} \)

16. A spring has a natural length of 30 cm. A force of 25 N is required to hold the spring at a length of 40 cm.

(I) What is the value of the proportionality constant \( k \) in Hooke’s Law? Answer: (d).

(a) 2.5  
(b) \( \frac{5}{2} \)  
(c) 125  
(d) 250  
(e) 30  
(f) 40  
(g) None of these.

(II) How much work is required to stretch the spring from 35 cm to 45 cm? Answer: (c).

(a) 2 Joules  
(b) 3 Joules  
(c) 2.5 Joules  
(d) 3.5 Joules  
(e) 4 Joules  
(f) 4.5 Joules  
(g) None of these.

17. A spring has a natural length of 30 in. If it requires 15 ft-lbs of work to stretch the spring to a length of 40 in, how much force is required to hold it stretched to a length of 45 in?

Answer: (a). \( 15 = \int_{0}^{10/12} kx \) dx, so \( k = 144 \times 15/50 = 43.2 \). Force = \( kx = 43.2 \times 15/12 = 54 \) lbs.

(a) 54 lbs  
(b) 45 lbs  
(c) 35 lbs  
(d) 37 lbs  
(e) 56 lbs  
(f) 29 lbs  
(g) None of these.

18. A trough is 8 feet tall, 5 feet across the top and 12 feet long. It is filled with water to a depth of 3 feet. The variable \( x \) measures distance from the vertex at bottom of the tank. The density of water is 62.5 lb/ft\(^3\).

(I) Which integral gives the work necessary to pump the water out of the trough? Answer: (c)

(a) 468.75 \( \int_{0}^{3} x^2 \) dx  
(b) 468.75 \( \int_{0}^{8} x^2 - 8x \) dx  
(c) 468.75 \( \int_{0}^{3} 8x - x^2 \) dx  
(d) 937.5 \( \int_{0}^{3} 3 - x \) dx  
(e) 937.5 \( \int_{0}^{3} x^2 - 8x \) dx  
(f) 468.75 \( \int_{0}^{3} 3x - x^2 \) dx  
(g) 937.5 \( \int_{0}^{8} 8x - x^2 \) dx  
(h) 937.5 \( \int_{0}^{3} \sqrt{x^2 - 8x} \) dx  
(i) 468.75 \( \int_{0}^{3} \sqrt{3x - x^2} \) dx  
(j) 937.5 \( \int_{0}^{8} \sqrt{8x - x^2} \) dx

(II) What are the units for the integral above? Answer: (e) None of these - should be ft-lbs

(a) Lbs  
(b) Newtons  
(c) Kilograms  
(d) Joules  
(e) None of these.
19. A tank in the shape of an inverted cone is 8 feet tall and 5 feet across the top. It is filled with water to a depth of 3 feet. The variable $x$ measures distance from the vertex at the bottom of the tank. The density of water is 62.5 lb/ft$^3$.

(I) Which integral gives the work necessary to pump the water out of the tank? **Answer:** (b)

(a) $6.10\pi \int_0^8 8x^2 - x^3 \, dx$  
(b) $6.10\pi \int_0^3 8x^2 - x^3 \, dx$  
(c) $6.10\pi \int_0^3 x^3 \, dx$  
(d) $24.4\pi \int_0^8 x^2 \, dx$

(e) $24.4\pi \int_0^8 x^2 - 8x \, dx$  
(f) $6.10\pi \int_0^3 8x - x^2 \, dx$  
(g) $6.10\pi \int_0^3 x^3 - 8x^2 \, dx$

(h) $24.4\pi \int_0^8 \sqrt{x^2 - 8x} \, dx$  
(i) $6.5\pi \int_0^3 \sqrt{8x - x^2} \, dx$  
(j) $16.2\pi \int_0^3 \sqrt{x^3 - 8x^2} \, dx$

(II) What are the units for the integral above? **Answer:** (d)

(a) Lbs  
(b) Newtons  
(c) Kilograms  
(d) Ft-Lbs.  
(e) Joules  
(f) None of these.

20. A cylindrical tank lying on its side has diameter 8 meters and length 12 meters. It is filled with water to a depth of 3 meters. The variable $x$ measures distance from the bottom of the tank. The density of water is 9800 N/m$^3$.

(I) Which integral gives the work necessary to pump the water out of the tank? **Answer:** (a)

(a) $235200 \int_3^0 (8 - x) \sqrt{8x - x^2} \, dx$  
(b) $235200 \int_0^8 (8 - x) \sqrt{8x - x^2} \, dx$  
(c) $117600 \int_3^0 (10 - x) \sqrt{10x - x^2} \, dx$  
(d) $117600 \int_3^0 (8 - x) \sqrt{(4 - x)^2 - 16} \, dx$  
(e) $235200 \int_0^3 (3 - x) \sqrt{3x - x^2} \, dx$

(f) $117600 \int_3^0 (x - 8) \sqrt{x^2 - 8x} \, dx$  
(g) $235200 \int_0^3 (4 - x) \sqrt{4x - x^2} \, dx$

(II) What are the units for the integral above? **Answer:** (e)

(a) Lbs  
(b) Newtons  
(c) Kilograms  
(d) Ft-Lbs.  
(e) Joules  
(f) None of these.

21. A 4 kg mass is hanging on a 30 meter rope as shown. The rope has a mass of 15 kgs. Which integral gives the work done in lifting the rope 10 meters? (Note that $x$ measures the length of the rope.) **Answer:** (iii)

(i) $\int_0^{10} 4.9x + 4 \, dx$  
(ii) $\int_0^{10} 4.9x + 39.2 \, dx$  
(iii) $\int_{20}^{30} 4.9x + 39.2 \, dx$  
(iv) $\int_{20}^{30} 4.9x + 19.6 \, dx$

(v) $\int_{20}^{30} 0.5x^2 + 2 \, dx$  
(vi) $\int_0^{10} 19.6x + 39.2, \, dx$  
(vii) $\int_{20}^{30} 4.9x + 39.2 \, dx$  
(viii) $\int_0^{10} 0.5x + 2 \, dx$

22. What are the units for the answer above? **Answer:** (e) Joules

(a) Lbs  
(b) Newtons  
(c) Kilograms  
(d) Ft-Lbs.  
(e) Joules  
(f) None of these.
23. A 4 lb weight is hanging on a 30 foot chain as shown. The chain weighs 15 lbs. What is the work done in lifting the weight 10 feet? (Note that $x$ measures the length of the chain.)  

Answer: (a)

(a) 165  (b) 115  (c) 65  (d) 195  (e) 291  (f) 125

24. What are the units for the answer above?  

Answer: (d)

(a) Lbs  (b) Newtons  (c) Kilograms  (d) Ft-Lbs.  (e) Joules  (f) None of these.

25. Which of the following integrals gives the length of the curve $y = \cos(3x^2)$ from $x = 0$ to $x = 2$?  

Answer: (h)

None of these. Should be $\int_0^2 \sqrt{1 + 36x^2 \sin^2(3x^2)} \, dx$

(a) $\int_0^2 \sqrt{1 + 6 \sin^2(3x^2)} \, dx$  (b) $\int_0^2 \sqrt{1 + \sin^2(6x)} \, dx$  (c) $\int_0^2 \sqrt{1 + 36 \sin^2(3x^2)} \, dx$  (d) $\int_0^2 \sqrt{1 + 36 \sin^2(6x)} \, dx$

(e) $\int_0^2 \sqrt{1 + 36x^2 \cos^2(3x^2)} \, dx$  (f) $\int_0^2 \sqrt{1 + 6 \sin^2(36x^2)} \, dx$  (g) $\int_0^2 \sqrt{1 + \cos^2(3x^2)} \, dx$  (h) None of these.

26. Which of the following integrals gives the length of the curve $y = e^{x^2}$ from $x = 0$ to $x = 1$?  

Answer: (b)

(a) $\int_0^1 \sqrt{1 + e^{2x}} \, dx$  (b) $\int_0^1 \sqrt{1 + 4x^2 e^{2x^2}} \, dx$  (c) $\int_0^1 \sqrt{1 + x^2 e^{x^2} - 1} \, dx$  (d) $\int_0^1 \sqrt{1 + (x^2 e^{x^2} - 1)^2} \, dx$

(e) $\int_0^1 \sqrt{1 + e^{2x^2}} \, dx$  (f) $\int_0^1 \sqrt{1 + 2xe^{2x^2}} \, dx$  (g) $\int_0^1 \sqrt{1 + (e^{x^2})^2} \, dx$  (h) None of these.