1. To find \( \int x^2 \sqrt{16 - x^2} \, dx \) by trig substitution, the first step you should make is to let:
   
   (a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)   
   (b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)   
   (c) \( x = 4 \tan \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)   
   (d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)   
   (e) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)   
   (f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)   
   (g) none of these

2. To find \( \int \frac{x^3}{\sqrt{x^2 - 16}} \, dx \) by trig substitution, the first step you should make is to let:
   
   (a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)   
   (b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)   
   (c) \( x = \sin \theta \) and \( dx = \cos \theta \, d\theta \)   
   (d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)   
   (e) \( x = \sec \theta \) and \( dx = \sec \theta \tan \theta \, d\theta \)   
   (f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)   
   (g) none of these

3. To find \( \int \frac{\sqrt{x^2 + 16}}{x^7} \, dx \) by trig substitution, the first step you should make is to let:
   
   (a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)   
   (b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)   
   (c) \( x = \sin \theta \) and \( dx = \cos \theta \, d\theta \)   
   (d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)   
   (e) \( x = \sec \theta \) and \( dx = \sec \theta \tan \theta \, d\theta \)   
   (f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)   
   (g) none of these

4. To find \( \int x^3 \sqrt{9 - x^2} \, dx \) by trig substitution, we let \( x = 3 \sin \theta \) and \( dx = 3 \cos \theta \, d\theta \). After simplifying, the integral becomes:
   
   (a) \( \int 243 \sin^3 \theta \cos^2 \theta \, d\theta \)   
   (b) \( \int 9 \sin^3 \theta \cos \theta \, d\theta \)   
   (c) \( \int 27 \sin^2 \theta \cos^2 \theta \, d\theta \)   
   (d) \( \int 243 \sin^3 \theta \cos \theta \, d\theta \)   
   (e) \( \int 81 \sin^3 \theta \sqrt{9 - \sin^2 \theta} \, d\theta \)   
   (f) \( \int 27 \sin^3 \theta \sqrt{9 - 9 \sin^2 \theta} \, d\theta \)   
   (g) none of these

5. To find \( \int x^2 \sqrt{9 + x^2} \, dx \) by trig substitution, we let \( x = 3 \tan \theta \) and \( dx = 3 \sec^2 \theta \, d\theta \). After simplifying, the integral becomes:
   
   (a) \( \int 243 \tan^2 \theta \sec^2 \theta \, d\theta \)   
   (b) \( \int 27 \tan^3 \theta \sec \theta \, d\theta \)   
   (c) \( \int 81 \tan^2 \theta \sec^3 \theta \, d\theta \)   
   (d) \( \int 81 \sec^3 \theta \tan^3 \theta \, d\theta \)   
   (e) \( \int 81 \tan^3 \theta \sqrt{9 + 9 \tan^2 \theta} \, d\theta \)   
   (f) \( \int 27 \tan^3 \theta \sqrt{9 + 9 \tan^2 \theta} \, d\theta \)   
   (g) none of these

6. To find \( \int \cos^6 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:
   
   (a) \( \cos^6 \theta = \cos^5 \theta \cos \theta \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)   
   (b) \( \sin^8 \theta = \sin^7 \theta \sin \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (c) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \)   
   (d) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)   
   (e) \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (f) \( \sin^2 \theta = 1 - \cos^2 \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (g) \( \cos^6 \theta = \cos^5 \theta \cos \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (h) \( \sin^8 \theta = \sin^7 \theta \sin \theta \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)

7. To find \( \int \cos^7 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:
   
   (a) \( \cos^7 \theta = \cos^6 \theta \cos \theta \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)   
   (b) \( \sin^8 \theta = \sin^7 \theta \sin \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (c) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \)   
   (d) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)   
   (e) \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (f) \( \sin^2 \theta = 1 - \cos^2 \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (g) \( \cos^7 \theta = \cos^6 \theta \cos \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)   
   (h) \( \sin^8 \theta = \sin^7 \theta \sin \theta \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)

8. Find \( \int \cos^3 \theta \sin^8 \theta \, d\theta \)
9. To find \( \int \sec^4 \theta \tan^4 \theta \ d\theta \) you should transform this integral into:

(a) \( \int \sec^2 \theta \tan^3 \theta \sec^2 \theta \ d\theta = \int \sec^2 \theta (\sec^2 \theta - 1)^2 \sec^2 \theta \ d\theta = \int \sec^8 \theta - 2 \sec^6 \theta + \sec^4 \theta \ d\theta \)

(b) \( \int \sec^2 \theta \tan^4 \theta \sec^2 \theta \ d\theta = \int (1 + \tan^2 \theta) \tan^4 \theta \sec^2 \theta \ d\theta = \int (\tan^4 \theta + \tan^6 \theta) \sec^2 \theta \ d\theta \)

(c) \( \int \sec^3 \theta \tan^3 \theta \sec \theta \tan \theta \ d\theta = \int \sec^2 \theta \sec \theta \tan \theta \sec \theta \tan \theta \ d\theta = \int \sec^2 \theta (\sec \theta \tan \theta)^2 \sec \theta \tan \theta \ d\theta \)

(d) \( \int \sec^2 \theta \tan^4 \sec^2 \theta \ d\theta = \int (1 + \tan^2 \theta)(\sec^2 \theta - 1) \sec^2 \theta \ d\theta \)

(e) \( \int \sec^2 \theta \tan^2 \sec^2 \theta \ d\theta = \int (1 + \tan^2 \theta)(\sec^2 \theta - 1) \sec \theta \tan \theta \ d\theta \)

(f) \( \int (1 + \tan^2 \theta)^2 (\sec^2 \theta - 1) \ d\theta \)

10. Find \( \int \sec^4 \theta \tan^4 \theta \ d\theta \).

11. Find the correct standard form of the partial fractions decomposition for \( \frac{x^3 + x + 200}{(x^2 + x - 2)(x - 1)(x^2 + 4)^2} \). Do not find the constants.

(a) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2} \)

(b) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3} + \frac{E}{x^2 + 4} + \frac{F}{(x^2 + 4)^2} \)

(c) \( \frac{Ax + B}{x^2 + x - 2} + \frac{C}{x - 1} + \frac{D}{(x - 1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2} \)

(d) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3} + \frac{Ex}{x^2 + 4} + \frac{F}{(x^2 + 4)^2} \)

(e) \( \frac{Ax + B}{x^2 + x - 2} + \frac{Cx + D}{(x - 1)^2} + \frac{Ex + F}{(x - 1)^3} + \frac{Gx}{x^2 + 4} + \frac{H}{(x^2 + 4)^2} \)

(f) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{Cx + D}{(x - 1)^2} + \frac{Ex + F}{(x - 1)^3} + \frac{G}{x^2 + 4} + \frac{H}{(x^2 + 4)^2} \)

12. In the partial fraction expansion \( \frac{x}{(x - 1)(x - 4)^2(x + 2)} = \frac{A}{x - 1} + \frac{B}{(x - 4)^2} + \frac{C}{x - 4} + \frac{D}{x + 2} \), the value of \( A \) is

(a) 1/81
(b) 1/3
(c) -1/3
(d) 1/9
(e) 2/9
(f) none of these

13. Given that the partial fractions expansion of \( \frac{x^3}{(x - 1)^2(x^2 + 1)} \) is \( \frac{1}{2} \frac{1}{(x - 1)^2} + \frac{1}{x - 1} - \frac{3}{2} \frac{x}{x^2 + 1} \), find \( \int \frac{x^3}{(x - 1)^2(x^2 + 1)} \ dx \)

14. (A) Find the partial fractions expansion of \( \frac{x + 7}{x^2 - x - 2} \).

(B) Evaluate \( \int \frac{x + 7}{x^2 - x - 2} \ dx \) using partial fractions.

15. To find \( \int \frac{2x + 1}{\sqrt{x^2 + x}} \ dx \), choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______

(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______

(C) Use trig substitution with \( x = \) _______ and \( dx = \) _______

(D) None of the three methods is needed. It can be done directly without them.

Evaluate the integral and show your work.
16. To find \( \int \frac{y + 1}{\sqrt{y}} \, dy \), choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______
(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______
(C) Use trig substitution with \( x = \) _______ and \( dx = \) _______
(D) None of the three methods is needed. It can be done directly without them.

Evaluate the integral and show your work

17. To find \( \int x^6 \sqrt{x^2 + 1} \, dx \), choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______
(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______
(C) Use trig substitution with \( x = \) _______ and \( dx = \) _______
(D) None of the three methods is needed. It can be done directly without them.

Do not evaluate the integral

18. To find \( \int \frac{t^3}{t^4 + 11} \, dt \) choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______
(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______
(C) Use trig substitution with \( t = \) _______ and \( dt = \) _______
(D) None of the three methods is needed. It can be done directly without them.

Evaluate the integral and show your work

19. To find \( \int s^4 + 11 \, ds \) choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______
(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______
(C) Use trig substitution with \( s = \) _______ and \( ds = \) _______
(D) None of the three methods is needed. It can be done directly without them.

Evaluate the integral and show your work

20. To find \( \int x^8 \sqrt{x^2 - 1} \, dx \), choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______
(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______
(C) Use trig substitution with \( x = \) _______ and \( dx = \) _______
(D) None of the three methods. It can be done directly without them.

Do not evaluate the integral.

21. To find \( \int \ln z \, dz \) choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______
(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______
(C) Use trig substitution with \( z = \) _______ and \( dz = \) _______
(D) None of the three methods is needed. It can be done directly without them.

Evaluate the integral and show your work

22. To find \( \int_0^2 xe^x \, dx \) choose the appropriate method and fill in the blanks.

(A) Use substitution with \( u = \) _______ and \( du = \) _______
(B) Use parts with \( u = \) _______ \( dv = \) _______ \( du = \) _______ and \( v = \) _______
(C) Use trig substitution with \( x = \) _______ and \( dx = \) _______
(D) None of the three methods is needed. It can be done directly without them.

Evaluate the integral and show your work
23. To find $\int re^r \, dr$ choose the appropriate method and fill in the blanks.

(A) Use substitution with $u = \underline{\hspace{2cm}}$ and $du = \underline{\hspace{2cm}}$

(B) Use parts with $u = \underline{\hspace{2cm}}$ $dv = \underline{\hspace{2cm}}$, $du = \underline{\hspace{2cm}}$ and $v = \underline{\hspace{2cm}}$

(C) Use trig substitution with $r = \underline{\hspace{2cm}}$ and $dr = \underline{\hspace{2cm}}$

(D) None of the three methods is needed. It can be done directly without them.

Evaluate the integral and show your work

24. To find $\int \theta^3 \cos \theta \, d\theta$ choose the appropriate method and fill in the blanks.

(A) Use substitution with $u = \underline{\hspace{2cm}}$ and $du = \underline{\hspace{2cm}}$

(B) Use parts with $u = \underline{\hspace{2cm}}$ $dv = \underline{\hspace{2cm}}$, $du = \underline{\hspace{2cm}}$ and $v = \underline{\hspace{2cm}}$

(C) Use trig substitution with $\theta = \underline{\hspace{2cm}}$ and $d\theta = \underline{\hspace{2cm}}$

(D) None of the three methods. It can be done directly without them.

Do not evaluate the integral.

25. Use (a) Simpson’s rule, (b) the trapezoidal rule, (c) a left end point Riemann sum, and (d) a right end point Riemann sum all with 6 subintervals to estimate $\int_1^4 f(x) \, dx$ using the function values from the table.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2.5</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>3.5</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

26. Use (a) Simpson’s rule and (b) the trapezoidal rule with 4 subintervals to estimate $\int_1^4 \frac{1}{x} \, dx$.

27. The distances across a lake are 8, 70, 60, 45, 10, and 5 meters as shown. They were measured at 6 meter intervals. Use (I) the Trapezoidal Rule and (II) Simpson’s rule to estimate the area of the lake.

(I) The Trapezoidal Rule gives:

(a) 1164 (b) 388 (c) 594 (d) 990 (e) 1140 (f) 995

(II) Simpson’s Rule gives:

(a) 628 (b) 1980 (c) 594 (d) 990 (e) 1140 (f) 1256

28. Let $g$ be a function on $[1, 4]$ such that $-6 \leq g^{(4)}(x) \leq 3$ for $1 \leq x \leq 4$.

(I) In the error formula for Simpson’s Rule, what is the best value of $K$?

(a) 0 (b) 4 (c) -6 (d) 6 (e) 5 (f) 1 (g) none of these.

(II) What is the smallest number of subintervals one should use in order to assure that Simpson’s Rule approximates $\int_1^4 g(x) \, dx$ to within $10^{-4}$?

(a) 25 (b) 13 (c) 8100 (d) 16 (e) 17 (f) 18 (g) none of these.

29. Let $f$ be a function on $[1, 4]$ such that $-1 \leq f''(x) \leq 5$ for all $x$ in $[1, 4]$. What is the smallest number of subintervals one should use in order to assure that the Trapezoidal Rule approximates $\int_1^4 f(x) \, dx$ to within $10^{-4}$?

(a) 1125 (b) 336 (c) 112 (d) 11,250 (e) 335 (f) 1060 (g) none of these.
30. The graph of $f(x)$ is shown.

Let $I$ be the exact value of the integral $\int_a^b f(x) \, dx$.
Let $L$ be the left-hand Riemann sum for $f$ with 58 subintervals.
Let $R$ be the right-hand Riemann sum for $f$ with 58 subintervals.
Let $M$ be the midpoint Riemann sum for $f$ with 58 subintervals.
Let $T$ be the trapezoidal rule for $f$ with 58 subintervals.

Write these five values in order starting with the smallest on the left.