Math 182 Review for test #2 Spring 2011.

1. Exactly two of the following are true about the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \). Mark only those two.
   
   (a) It converges absolutely by the ratio test.
   (b) It converges by the ratio test.
   (c) It diverges by the ratio test.
   (d) It converges absolutely because a \( p \)-series with \( p < 1 \) converges.
   (e) It converges conditionally.
   (f) It converges because it is absolutely convergent and an absolutely convergent series is also convergent.
   (g) It converges by the Alternating Series test.
   (h) It diverges because it’s a \( p \)-series with \( p < 1 \).
   (i) It converges by the limit comparison test.
   (j) It diverges by the comparison test.

2. Exactly three of the following are true about the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^3}} \). Mark only those three.
   
   (a) It converges absolutely by the ratio test.
   (b) It converges by the ratio test.
   (c) It diverges by the ratio test.
   (d) It converges absolutely because a \( p \)-series with \( p > 1 \) converges.
   (e) It converges conditionally.
   (f) It converges because it is absolutely convergent and an absolutely convergent series is also convergent.
   (g) It converges by the Alternating Series test.
   (h) It diverges because it’s a \( p \)-series with \( p > 1 \).
   (i) It diverges by the limit comparison test.

3. Exactly one of the following is true about the series \( \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^3} \). Mark only that one.
   
   (a) It converges by the ratio test.
   (b) It diverges by the ratio test.
   (c) It converges by the Alternating Series test.
   (d) It converges by the comparison test because \( 0 \leq \frac{\sin^2 n}{n^3} \leq \frac{1}{n^3} \).
   (e) It diverges by the comparison test because \( \frac{\sin^2 n}{n^3} \geq \frac{1}{n^3} \).
4. Exactly two of the following are true about the series \( \sum_{n=1}^{\infty} \frac{n}{3^n} \). Mark **only** those two.

(a) It converges absolutely by the ratio test.
(b) It converges by the ratio test.
(c) It diverges by the ratio test.
(d) It converges conditionally.
(e) It converges by the Alternating Series test.
(f) It diverges because it’s a \( p \)-series with \( p > 1 \).
(g) It diverges by the comparison test.
(h) It diverges because it is geometric with \( r = 3 \).
(i) It converges because it is geometric with \( r = \frac{1}{3} \).

5. Exactly one of the following is true about the series \( \sum_{n=1}^{\infty} (-1)^n \frac{n!}{2^n} \). Mark **only** that one.

(a) It converges absolutely by the ratio test.
(b) It converges by the ratio test.
(c) It diverges by the ratio test.
(d) The ratio test is inconclusive for this series.
(e) It converges conditionally.
(f) It converges by the Alternating Series test.
(g) It diverges because it’s a \( p \)-series with \( p > 1 \).
(h) It diverges because it is geometric with \( r > 1 \).
(i) It converges because it is geometric with \( r < 1 \).

6. If you apply the limit comparison test to \( \sum 2\sqrt{n} + 7 \), the series you should use for comparison is

(a) \( \sum \frac{1}{n^2} \)  
(b) \( \sum \frac{1}{n^2} \)  
(c) \( \sum n^2 \)  
(d) \( \sum \frac{1}{n^2} \)  
(e) \( \sum \frac{1}{3n} \)  
(g) none of these

7. Apply the limit comparison test to \( \sum \frac{2\sqrt{n} + 7}{3n + 5} \) to decide whether it converges or diverges.

8. Apply the integral test to determine if the series \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} \) converges or diverges.

9. Apply the integral test to determine if the series \( \sum_{n=3}^{\infty} \frac{1}{n \ln n} \) converges or diverges.

10. Circle the correct answer (a), (b) or (c): The series \( \sum \frac{(-1)^n+12n^2}{5n+1} \) (a) converges absolutely. (b) converges conditionally. (c) diverges. Apply the appropriate test (or tests) to justify your answer, and give the names of those tests.
11. Circle the correct answer (a), (b) or (c): The series \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}3^n}{n!} \) (a) converges absolutely. (b) converges conditionally. (c) diverges. Apply the appropriate test (or tests) to justify your answer, and give the names of those tests.

12. Circle the correct answer (a), (b) or (c): The series \( \sum (-1)^n \frac{2n^2}{3 + 5n^2} \) (a) converges absolutely. (b) converges conditionally. (c) diverges. Apply the appropriate test (or tests) to justify your answer, and give the names of those tests.

13. Circle the correct answer (a), (b) or (c): The series \( \sum (-1)^n \frac{3^n}{\sqrt{n}} \) (a) converges absolutely. (b) converges conditionally. (c) diverges. Apply the appropriate test (or tests) to justify your answer, and give the names of those tests.

14. What is the solution to the inequality \(|2x - 1| < 3|\)

15. Apply the ratio test to find the largest open interval on which series \( \sum \frac{(-1)^n n}{8^{n+1}} (x - 2)^{3n} \) converges absolutely. Do not check the endpoints.

- (a) \(-2 < x < 0\)
- (b) \(-\frac{1}{2} < x < \frac{1}{2}\)
- (c) \(-8 < x < 8\)
- (d) \(-1 < x < 2\)
- (e) \(-2 < x < 2\)
- (f) \(-\frac{1}{5} < x < \frac{1}{5}\)
- (g) \(0 < x < 4\)
- (h) none of these.

16. Use your answer in the previous problem to find the radius of convergence of \( \sum \frac{(-1)^n n}{8^{n+1}} (x - 2)^{3n} \).

- (a) \(\frac{1}{8}\)
- (b) \(4\)
- (c) \(\frac{1}{2}\)
- (d) \(1\)
- (e) \(-\frac{1}{2}\)
- (f) \(8\)
- (g) \(\frac{1}{4}\)
- (h) none of these.

17. If \(|x| < 1\), \( \sum_{n=0}^{\infty} x^n = \)

- (a) \(e^x\)
- (b) \(\frac{x}{1 - x}\)
- (c) \(\cos x\)
- (d) \(\ln x\)
- (e) \(\frac{1}{1 - x}\)
- (f) None of these.

18. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n = \)

- (a) \(e^x\)
- (b) \(\sin x\)
- (c) \(\cos x\)
- (d) \(\ln x\)
- (e) \(\frac{1}{1 - x}\)
- (f) None of these.

19. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} n^{2n} = \)

- (a) \(0\)
- (b) \(1\)
- (c) \(-1\)
- (d) \(e\)
- (e) \(\frac{1}{e}\)
- (f) \(\sqrt{e}\)
- (g) \(e^2\)
- (f) None of these.

20. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)!} n^{2n+1} = \)

- (a) \(0\)
- (b) \(1\)
- (c) \(-1\)
- (d) \(e\)
- (e) \(\frac{1}{e}\)
- (f) \(\sqrt{e}\)
- (g) \(e^2\)
- (f) None of these.
21. \[\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} =\]
(a) 0 (b) 1 (c) -1 (d) \(\frac{1}{e}\) (e) \(\sqrt{e}\) (g) \(e^2\) (f) None of these.

22. \[\sum_{n=0}^{\infty} \frac{1}{2^n n!} =\]
(a) 0 (b) 1 (c) -1 (d) \(\frac{1}{e}\) (f) \(\sqrt{e}\) (g) \(e^2\) (f) None of these.

23. \[\sum_{n=0}^{\infty} \frac{2^n}{n!} =\]
(a) 0 (b) 1 (c) -1 (d) \(\frac{1}{e}\) (f) \(\sqrt{e}\) (g) \(e^2\) (f) None of these.

24. Evaluate the following argument by circling whether each line is correct or not.
(a) \(\sqrt{n} + n \leq n^2 + n^2 = 2n^2\). (correct or not?)
(b) Therefore \(\frac{1}{\sqrt{n} + n} \geq \frac{1}{2n^2}\). (correct or not?)
(c) But \(\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^2}\) is convergent because it is one-half times a convergent \(p\)-series \((p = 2)\). (correct or not?)
(d) Lines (a) - (c) show that the series \(\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n}\) converges by the comparison test. (correct or not?)

25. Evaluate the following argument by circling whether each line is correct or not.
(a) \(\sqrt{n} + n \leq n + n = 2n\). (correct or not?)
(b) Therefore \(\frac{1}{\sqrt{n} + n} \geq \frac{1}{2n}\). (correct or not?)
(c) But \(\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n}\) is divergent because it is one-half times the divergent harmonic series. (correct or not?)
(d) Lines (a) - (c) show that the series \(\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n}\) diverges by the comparison test. (correct or not?)

26. Evaluate the following argument by circling whether each line is correct or not.
(a) If you divide \(\frac{1}{\sqrt{n} + n^2}\) by \(\frac{1}{n^2}\) you get \(\frac{n^2}{\sqrt{n} + n^2}\). (correct or not?)
(b) \(\lim_{n \to \infty} \frac{n^2}{\sqrt{n} + n^2} = 1\). (correct or not?)
(c) But \(\sum_{n=1}^{\infty} \frac{1}{n^2}\) is convergent because it is a \(p\)-series with \((p > 1)\). (correct or not?)
(d) Lines (a) - (c) show that the series \(\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n^2}\) converges by the limit comparison test. (correct or not?)

27. \[\sum_{n=0}^{\infty} (-1)^n \frac{2^{n-1}}{7n+2}\] equals which of these? **On the exam you will need to show your work.**
(a) \(-\frac{1}{441}\) (b) \(-\frac{1}{245}\) (c) \(-\frac{2}{441}\) (d) \(\frac{1}{126}\) (e) \(\frac{7}{18}\) (f) \(\frac{1}{70}\) (g) None of these - it diverges.
28. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{3n+2} \) equals which of these? **On the exam you will need to show your work.**

   (a) \( \frac{-1}{12} \)  
   (b) \( \frac{1}{84} \)  
   (c) \( \frac{1}{9} \)  
   (d) \( \frac{1}{63} \)  
   (e) \( \frac{4}{3} \)  
   (f) \( \frac{-4}{3} \)  
   (g) None of these - it diverges

29. Suppose that \( \sum_{n=1}^{\infty} \frac{n}{3^n} = \frac{3}{4} \). What is \( \sum_{n=3}^{\infty} \frac{n}{3^n} \)? **On the exam you will need to show your work.**

   (a) \( \frac{5}{36} \)  
   (b) \( \frac{9}{34} \)  
   (c) \( \frac{2}{3} \)  
   (d) \( \frac{1}{12} \)  
   (e) \( \frac{7}{34} \)  
   (f) \( \frac{7}{36} \)  
   (g) None of these.

30. Which of the following is the Maclaurin series for \( \cos(x^2) \)?

   (a) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n+1)!} \)  
   (b) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n} \)  
   (c) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n} \)  
   (d) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n} \)

31. Which of the following is the Maclaurin series for \( xe^x \)?

   (a) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n!} \)  
   (b) \( \sum_{n=0}^{\infty} \frac{1}{n!} x^{n+1} \)  
   (c) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{n-1} \)  
   (d) \( \sum_{n=0}^{\infty} \frac{1}{n} x^{n+1} \)

32. Show how the Maclaurin series for \( e^x \) is used to find \( \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} \).

33. Show how the Maclaurin series for \( \cos x \) is used to find \( \lim_{x \to 0} \frac{\cos x - \frac{x^4}{24} + \frac{x^6}{2} - 1}{x^6} \).

34. Which of the following is the partial sum of the first 4 terms of the Taylor series for \( f(x) = \sqrt{x} \) about \( a = 4 \)? **On the exam you will need to show your work.**

   (a) \( 1 - \frac{1}{4} x + \frac{1}{32} x^2 - \frac{1}{512} x^3 \)  
   (b) \( 2 + \frac{1}{4} x - \frac{1}{64} x^2 + \frac{1}{512} x^3 \)  
   (c) \( 4 + \frac{1}{2} x - \frac{1}{32} x^2 + \frac{6}{256} x^3 \)  
   (d) \( 1 - \frac{1}{4} (x-4) + \frac{1}{32} (x-4)^2 - \frac{1}{512} (x-4)^3 \)  
   (e) \( 2 + \frac{1}{4} (x-4) - \frac{1}{64} (x-4)^2 + \frac{1}{512} (x-4)^3 \)  
   (f) \( 2 - \frac{1}{4} (x-4) + \frac{1}{64} (x-4)^2 - \frac{3}{256} (x-4)^3 \)  
   (g) \( 4 + \frac{1}{2} (x-4) - \frac{1}{32} (x-4)^2 + \frac{6}{256} (x-4)^3 \)  
   (h) \( 1 + \frac{1}{3} (x-4) - \frac{2}{9} (x-4)^2 + \frac{10}{27} (x-4)^3 \)
35. Which of the following is the partial sum of the first 4 terms of the Taylor series for \( f(x) = \frac{1}{\sqrt{x}} \) about \( a = 1 \)? On the exam you will need to show your work.

(a) \( 1 - \frac{2}{3}(x-1) + \frac{5}{9}(x-1)^2 - \frac{40}{81}(x-1)^3 \)
(b) \( 1 - \frac{1}{3}(x-1) + \frac{2}{9}(x-1)^2 - \frac{14}{81}(x-1)^3 \)
(c) \( 1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3 \)
(d) \( 1 + \frac{1}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{5}{81}(x-1)^3 \)
(e) \( 1 - \frac{1}{3}(x-1) + \frac{8}{9}(x-1)^2 - \frac{28}{27}(x-1)^3 \)
(f) \( 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 \)
(g) \( 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 \)
(h) \( 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 \)

36. Which of the following is the partial sum of the first 4 terms of the Maclaurin series for \( f(x) = \sqrt[4]{x} + 1 \). On the exam you will need to show your work.

(a) \( 1 + \frac{3}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{5}{128}(x-1)^3 \)
(b) \( 1 + \frac{1}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{7}{128}(x-1)^3 \)
(c) \( 1 - \frac{3}{4}(x-1) + \frac{21}{32}(x-1)^2 - \frac{77}{128}(x-1)^3 \)
(d) \( 1 - \frac{1}{4}(x-1) + \frac{5}{32}(x-1)^2 - \frac{15}{128}(x-1)^3 \)
(e) \( 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \)
(f) \( 1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3 \)
(g) \( 1 + \frac{1}{4}x - \frac{3}{16}x^2 + \frac{21}{64}x^3 \)
(h) \( 1 - \frac{1}{4}x + \frac{5}{32}x^2 - \frac{15}{128}x^3 \)

37. Which of the following is the partial sum of the first 4 terms of the Maclaurin series for \( f(x) = \frac{1}{\sqrt{x+1}} \)? On the exam you will need to show your work.

(a) \( 1 + \frac{3}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{5}{128}(x-1)^3 \)
(b) \( 1 + \frac{1}{4}(x-1) - \frac{3}{32}(x-1)^2 + \frac{7}{128}(x-1)^3 \)
(c) \( 1 - \frac{3}{4}(x-1) + \frac{21}{32}(x-1)^2 - \frac{77}{128}(x-1)^3 \)
(d) \( 1 - \frac{1}{4}(x-1) + \frac{5}{32}(x-1)^2 - \frac{15}{128}(x-1)^3 \)
(e) \( 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \)
(f) \( 1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3 \)
(g) \( 1 - \frac{1}{4}x + \frac{5}{16}x^2 - \frac{45}{64}x^3 \)
(h) \( 1 - \frac{1}{4}x + \frac{5}{32}x^2 - \frac{15}{128}x^3 \)
In the following, circle “T” if it is always true and “F” otherwise.

1. T F If \( f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \), then \( c_n = \frac{f^{(n)}(x)}{n!} \).

2. T F If \( f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n \), then \( c_n = f^{(n)}(a) \).

3. T F If \( a_n > 0 \) for every \( n \) and \( \sum_{n=0}^{k} a_n \leq 8 \) for all \( k \) then \( \sum_{n=0}^{k} a_n \) converges.

4. T F \( \lim_{n \to \infty} \frac{n}{n+1} = 1 \), which is not zero, therefore it diverges by the \( n^{th} \) term divergence test.

5. T F If \( \sum_{n=1}^{\infty} c_n = 1 \) then it diverges by the \( n^{th} \) term divergence test.

6. T F If \( \{b_n\} \) is increasing and bounded above, then it must converge.

7. T F If \( \{b_n\} \) is increasing and bounded below, then it must converge.

8. T F \( \left\{\frac{1}{n}\right\} \) is a decreasing sequence.

9. T F \( \{\sin(n)\} \) is a monotonic sequence.

10. T F \( \left\{\frac{n}{2}\right\} \) is a monotonic sequence.

11. T F If \( \lim_{n \to \infty} a_n \neq 0 \) then it diverges.

12. T F If \( \lim_{n \to \infty} a_n = 0 \) then \( \lim_{n \to \infty} |a_n| = 0 \)

13. T F If \( \lim_{n \to \infty} |a_n| = 0 \) then \( \lim_{n \to \infty} a_n = 0 \)

14. T F If \( \lim_{n \to \infty} a_n = 0 \) then \( \{a_n\} \) converges.

15. T F If \( \lim_{n \to \infty} a_n = 0 \) then \( \sum a_n \) converges.

16. T F If \( \sum a_n \) converges then \( \lim_{n \to \infty} a_n = 0 \).

17. T F If \( \sum a_n \) converges, then \( \sum_{n=1}^{\infty} |a_n| \) converges.

18. T F If \( \sum_{n=1}^{\infty} |a_n| \) converges, then \( \sum_{n=1}^{\infty} a_n \) converges.

19. T F \( \left\{\frac{1}{n}\right\} \) is called the harmonic series.

20. T F \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges.

21. T F \( \left\{\frac{1}{n}\right\} \) diverges because it is the harmonic series.
22. T  F  $\sum_{n=1}^{\infty} \frac{1}{n}$ is decreasing.

23. T  F  If the sequence $\{a_n\}$ converges to 11, so does $\{a_{n+2}\}$.

24. T  F  $\sum_{n=1}^{\infty} r^n$ converges to $\frac{r}{1-r}$.

25. T  F  $\sum_{n=0}^{\infty} r^n$ converges to $\frac{1}{1-r}$ if $-1 < r < 1$.

26. T  F  $\sum_{n=0}^{\infty} r^n$ is the same as $\sum_{n=1}^{\infty} r^{n-1}$.

27. T  F  $\sum_{n=0}^{\infty} 5^n$ diverges.

28. T  F  $\sum_{n=0}^{\infty} 1.0001^n$ diverges.

29. T  F  $\sum_{n=0}^{\infty} 0.999^n$ diverges.

30. T  F  $\sum_{n=1}^{\infty} n^n$ is a geometric series.

31. T  F  According to the “sandwich (or squeeze) theorem”, if $\{b_n\}$ converges to 3 and $\{a_n\}$ converges to 3 and $b_n \leq c_n \leq a_n$ then $\{c_n\}$ converges to 3.

32. T  F  The “sandwich (or squeeze) theorem” shows that $\left\{\frac{\cos n}{n}\right\}$ converges to 0.

33. T  F  The “sandwich (or squeeze) theorem” shows that $\{n \cos n\}$ converges.

34. T  F  If $\sum_{n=1}^{k} a_n \geq 4$ for every positive integer $k$, then $\sum_{n=1}^{\infty} a_n$ converges.

35. T  F  If $\sum_{n=1}^{k} |a_n| \leq 201$ for every positive integer $k$, then $\sum_{n=1}^{\infty} a_n$ converges.

36. T  F  $\sum_{n=1}^{\infty} \frac{1}{n}$ converges to 0.

37. T  F  $\lim_{n \to \infty} \frac{n}{n+1} = 1$, therefore it diverges because the $n^{th}$ term does not go to 0.

38. T  F  $\lim_{n \to \infty} \frac{n}{n+1} = 1$, therefore $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges because the $n^{th}$ term does not go to 0.

39. T  F  $\sum_{n=1}^{\infty} 3^n = \frac{3}{1-3} = \frac{-3}{2}$.

40. T  F  $\sum_{n=1}^{\infty} r^{n+2} = \frac{r^3}{1-r}$ for $|r| < 1$. 