1. Let $R$ be the region bounded between the curves $y = x^2$ and $y = 3x$.

   I. Which integral below gives the volume of the solid obtained by revolving $R$ about the $x$–axis using the washer method? Answer: (f)
   
   II. Which integral below gives the volume of the solid obtained by revolving $R$ about the $x$–axis using the shell method? Answer: (p)
   
   III. Which integral below gives the volume of the solid obtained by revolving $R$ about the $y$–axis using the washer method? Answer: (q)
   
   IV. Which integral below gives the volume of the solid obtained by revolving $R$ about the $y$–axis using the shell method? Answer: (j) $2\pi \int_0^3 3x^2 - x^3 \, dx = \pi \int_0^3 6x^2 - 2x^3 \, dx$
   
   V. Which integral below gives the volume of the solid obtained by revolving $R$ about the line $x = 3$ using the shell method? Answer: (l) $2\pi \int_0^3 (3-x)(3x-x^2) \, dx = \pi \int_0^3 18x - 12x^2 + 2x^3 \, dx$
   
   VI. Which integral below gives the volume of the solid obtained by revolving $R$ about the line $y = 9$ using the washer method? Answer: (e) $\pi \int_0^3 (9-x^2)^2 - (9-3x)^2 \, dx = \pi \int_0^3 x^4 - 27x^2 + 54x \, dx$

2. Evaluate integrals (g) and (r) above.

   Answer: (g) $= 81\pi/10$ (r) $= 27\pi/10$
3. Circle the differential equations below that are separable. Answer: (a) (c) (d) (e).
   
   (a) \( \frac{dy}{dx} = xy \)  \hspace{0.5cm} (b) \( \frac{dy}{dx} = x + y \)  \hspace{0.5cm} (c) \( \frac{dy}{dx} = \ln y \)  \hspace{0.5cm} (d) \( \frac{dy}{dx} = xy^2 + x \cos(y) \)  \hspace{0.5cm} (e) \( \frac{dy}{dx} = e^x \)  \hspace{0.5cm} (f) \( \frac{dy}{dx} = \sin(yx) \)

4. Find the general solution to the differential equation \( y' = 2xy^2 \). Answer: (a)
   
   (a) \( y = -\frac{1}{x^2 + C} \)  \hspace{0.5cm} (b) \( y = e^{x^2 + C} \)  \hspace{0.5cm} (c) \( y = e^{2x^2 + C} \)  \hspace{0.5cm} (d) \( y = Ce^{x^2} \)  \hspace{0.5cm} (e) \( y = \frac{C}{x^2} \)  \hspace{0.5cm} (f) \( y = \frac{1}{x^2 - C} \)  \hspace{0.5cm} (g) none of these

5. Find the general solution to the differential equation \( \frac{dy}{dt} = 2ty + t \). Answer: (f)
   
   (a) \( y = Ce^{t^2} \)  \hspace{0.5cm} (b) \( y = Ce^{2t^2} - 1 \)  \hspace{0.5cm} (c) \( y = 2e^{t^2} - 1/2 \)  \hspace{0.5cm} (d) \( y = e^{x^2} - C/2 \)  \hspace{0.5cm} (e) \( y = Ce^{t^2} - 1/2 \)  \hspace{0.5cm} (f) \( y = e^{x^2} - C/2 \)  \hspace{0.5cm} (g) none of these

6. Solve \( y' = x^2 y, y(0) = 3 \). Answer: (c)
   
   (a) \( y = 3e^{x^3} \)  \hspace{0.5cm} (b) \( y = e^{3x^3/3} \)  \hspace{0.5cm} (c) \( y = 3e^{x^3/3} \)  \hspace{0.5cm} (d) \( y = 3 + \frac{x^3}{3} \)  \hspace{0.5cm} (e) \( y = e^{3x^3} \)  \hspace{0.5cm} (f) \( y = 3e^{3x^3} \)  \hspace{0.5cm} (g) none of these

7. Solve \( \frac{dy}{dt} = y^2 \sin t, y(0) = 3 \). Answer: (d)
   
   (a) \( y = \frac{1}{\sin t + 1/3} \)  \hspace{0.5cm} (b) \( y = \frac{\sin t}{\sqrt{\cos t - 2/3}} \)  \hspace{0.5cm} (c) \( y = 3e^{\cos t - 1} \)  \hspace{0.5cm} (d) \( y = e^{2\sin t/3} \)  \hspace{0.5cm} (e) \( y = \frac{3}{\cos t + 1} \)  \hspace{0.5cm} (f) \( y = \frac{3}{\cos t - 1} \)  \hspace{0.5cm} (g) none of these

8. A sample of radioactive material loses 30% of its mass in 5 days. What is its half life? Answer: (e)
   
   (a) 7 days \hspace{0.5cm} (b) 7.6 days \hspace{0.5cm} (c) 6.7 days \hspace{0.5cm} (d) 8.5 days \hspace{0.5cm} (e) 9.7 days \hspace{0.5cm} (f) 10 days \hspace{0.5cm} (g) 10.4 days

9. 80% of a sample of radioactive material remains after 11 days. How much remains after 13 days? Answer: (a)
   
   (a) 76.8% \hspace{0.5cm} (b) 87.6% \hspace{0.5cm} (c) 67.7% \hspace{0.5cm} (d) 75.3% \hspace{0.5cm} (e) 71.4% \hspace{0.5cm} (f) 69.1% \hspace{0.5cm} (g) 78.2%

10. Which of the following summarizes Newton’s law of cooling? Answer: (c)
    
    (a) The temperature of an object is proportional to the the surrounding temperature.
    
    (b) The temperature of an object is proportional to the difference between its temperature and the surrounding temperature.
    
    (c) The rate of change of the temperature of an object is proportional to the difference between its temperature and the surrounding temperature.
    
    (d) The rate of change of the temperature of an object is proportional to the surrounding temperature.
    
    (e) The difference between the temperature of an object and the surrounding temperature is proportional to the its temperature.

11. If \( H(t) \) is the temperature of an object at time \( t \) and \( H_S \) is the constant surrounding temperature, which of the following is Newton’s law of cooling? Answer: (b)
    
    (a) \( \frac{dH}{dt} = k(H - H_S) \) for some \( k > 0 \). \hspace{0.5cm} (b) \( \frac{dH}{dt} = k(H - H_S) \) for some \( k < 0 \). \hspace{0.5cm} (c) \( \frac{dH}{dt} = H - H_S \).
    
    (d) \( \frac{dH}{dt} = kH_S \) for some \( k > 0 \). \hspace{0.5cm} (e) \( \frac{dH}{dt} = kH_S \) for some \( k < 0 \). \hspace{0.5cm} (f) \( \frac{dH}{dt} = kH \) for some \( k \).
12. A mass of 2 grams is located at (3, 2), a mass of 3 grams is located at (−1, 4), and a mass of 7 grams is located at (2, 3).

(I) What are moments of this system about the $x$–axis and the $y$–axis? Answer: (e)

(a) $M_x = 23$ $M_y = 33$  
(b) $M_x = 17$ $M_y = 33$  
(c) $M_x = 17$ $M_y = 37$  
(d) $M_x = 37$ $M_y = 23$  
(e) $M_x = 37$ $M_y = 17$  
(f) $M_x = 5$ $M_y = 7$

(II) What is the centroid of this system? Answer: (c)

(a) $\bar{x} = \frac{33}{16}$, $\bar{y} = \frac{21}{16}$  
(b) $\bar{x} = \frac{77}{54}$, $\bar{y} = \frac{17}{54}$  
(c) $\bar{x} = \frac{17}{12}$, $\bar{y} = \frac{37}{12}$  
(d) $\bar{x} = \frac{31}{54}$, $\bar{y} = \frac{23}{54}$  
(e) $\bar{x} = \frac{5}{12}$, $\bar{y} = \frac{7}{12}$

13. A region is bounded by $y = f(x)$ between $x = a$ and $x = b$ in the first quadrant. It is known that $\int_a^b xf(x) \, dx = 20$, $\int_a^b f^2(x) \, dx = 16$, and $\int_a^b f(x) \, dx = 2$.

(I) Which choice below is the moment of the region about the $x$–axis? Answer: (h)

(a) $\frac{1}{2}$  
(b) $\frac{1}{4}$  
(c) $\frac{1}{10}$  
(d) $\frac{1}{16}$  
(e) 16  
(f) 2  
(g) 20  
(h) 8  
(i) 10  
(j) 4

(II) Which choice below is the moment of the region about the $y$–axis? Answer: (g)

(a) $\frac{1}{2}$  
(b) $\frac{1}{4}$  
(c) $\frac{1}{10}$  
(d) $\frac{1}{16}$  
(e) 16  
(f) 2  
(g) 20  
(h) 8  
(i) 10  
(j) 4

(III) Which choice below is the $x$–coordinate of centroid of the region ($\bar{x}$)? Answer: (i)

(a) $\frac{5}{2}$  
(b) $\frac{7}{2}$  
(c) $\frac{3}{2}$  
(d) $\frac{1}{2}$  
(e) $\frac{5}{2}$  
(f) $\frac{7}{2}$

(IV) Which choice below is the $y$–coordinate of centroid of the region ($\bar{y}$)? Answer: (j)

(a) $\frac{5}{2}$  
(b) $\frac{7}{2}$  
(c) $\frac{3}{2}$  
(d) $\frac{1}{2}$  
(e) $\frac{5}{2}$  
(f) $\frac{7}{2}$

14. Let $R$ be the region in the first quadrant bounded between $y = \sqrt{x}$, the $x$–axis and the line $x = 8$. What are moments of the region $R$ about the $x$–axis and the $y$–axis? Answer: (a)

(a) $M_x = \frac{48}{5}$, $M_y = \frac{384}{7}$  
(b) $M_x = \frac{7}{5}$, $M_y = \frac{38}{7}$  
(c) $M_x = \frac{8}{7}$, $M_y = \frac{34}{5}$  
(d) $M_x = \frac{351}{7}$, $M_y = \frac{98}{5}$  
(e) $M_x = \frac{36}{5}$, $M_y = \frac{291}{7}$  
(f) $M_x = \frac{384}{7}$, $M_y = \frac{48}{5}$

15. Let $R$ be the region in the first quadrant bounded between $y = x^3$, the $x$–axis and the line $x = 1$. What are the coordinates of the centroid of this region $R$? Answer: (e)

(a) $\bar{x} = \frac{5}{4}$, $\bar{y} = \frac{14}{5}$  
(b) $\bar{x} = \frac{2}{4}$, $\bar{y} = \frac{1}{5}$  
(c) $\bar{x} = \frac{5}{6}$, $\bar{y} = \frac{1}{6}$  
(d) $\bar{x} = \frac{4}{11}$, $\bar{y} = \frac{1}{5}$  
(e) $\bar{x} = \frac{5}{6}$, $\bar{y} = \frac{2}{3}$  
(f) $\bar{x} = \frac{5}{6}$, $\bar{y} = \frac{2}{3}$

16. A spring has a natural length of 30 cm. A force of 25 N is required to hold the spring stretched to a length of 40 cm.

(I) What is the value of the proportionality constant $k$ in Hooke’s Law? Answer: (d).

(a) 2.5  
(b) 2  
(c) 125  
(d) 250  
(e) 30  
(f) 40  
(g) None of these.

(II) How much work is required to stretch it from 35 cm to 45 cm? Answer: (c).

(a) 2 Joules  
(b) 3 Joules  
(c) 2.5 Joules  
(d) 3.5 Joules  
(e) 4 Joules  
(f) 4.5 Joules  
(g) None of these.

17. A spring has a natural length of 30 in. If it requires 15 ft-lbs of work to stretch the spring to a length of 40 in, how much force is required to hold it stretched to a length of 45 in?

Answer: (a). $15 = \int_0^{10/12} kx \, dx$, so $k = 144 \times \frac{15}{50} = 43.2$. Force $= kx = 43.2 \times \frac{15}{12} = 54$ lbs.

(a) 54 lbs  
(b) 45 lbs  
(c) 35 lbs  
(d) 37 lbs  
(e) 56 lbs  
(f) 29 lbs  
(g) None of these.
18. A trough is 8 feet tall, 5 feet across the top and 12 feet long. It is filled with water to a depth of 3 feet. The variable \( x \) measures distance from the vertex at bottom of the tank. The density of water is 62.5 lb/ft\(^3\).

(I) Which integral gives the work necessary to pump the water out of the trough? \( \text{Answer: (c)} \)

(a) \( 468.75 \int_0^3 x^2 \, dx \)  
(b) \( 468.75 \int_0^8 x^2 - 8x \, dx \)  
(c) \( 468.75 \int_0^8 8x - x^2 \, dx \)  
(d) \( 937.5 \int_0^3 3 - x \, dx \)  
(e) \( 937.5 \int_0^3 x^2 - 8x \, dx \)  
(f) \( 468.75 \int_0^3 3x - x^2 \, dx \)  
(g) \( 937.5 \int_0^8 8x - x^2 \, dx \)  
(h) \( 937.5 \int_0^3 \sqrt{x^2 - 8x} \, dx \)  
(i) \( 468.75 \int_0^8 \sqrt{3x - x^2} \, dx \)  
(j) \( 937.5 \int_0^8 \sqrt{8x - x^2} \, dx \)

(II) What are the units for the integral above? \( \text{Answer: (e) None of these - should be ft-lbs} \)

(a) Lbs  
(b) Newtons  
(c) Kilograms  
(d) Joules  
(e) None of these.

19. A tank in the shape of an inverted cone is 8 feet tall and 5 feet across the top. It is filled with water to a depth of 3 feet. The variable \( x \) measures distance from the vertex at bottom of the tank. The density of water is 62.5 lb/ft\(^3\).

(I) Which integral gives the work necessary to pump the water out of the tank? \( \text{Answer: (b)} \)

(a) \( 6.10 \pi \int_0^8 8x^2 - x^3 \, dx \)  
(b) \( 6.10 \pi \int_0^8 3x^2 - x^3 \, dx \)  
(c) \( 6.10 \pi \int_0^3 x^3 \, dx \)  
(d) \( 24.4 \pi \int_0^3 x^2 \, dx \)  
(e) \( 24.4 \pi \int_0^8 x^2 - 8x \, dx \)  
(f) \( 6.10 \pi \int_0^3 8x - x^2 \, dx \)  
(g) \( 6.10 \pi \int_0^3 x^3 - 8x^2 \, dx \)  
(h) \( 24.4 \pi \int_0^8 \sqrt{x^2 - 8x} \, dx \)  
(i) \( 6.5 \pi \int_0^8 \sqrt{8x - x^2} \, dx \)  
(j) \( 16.2 \pi \int_0^3 \sqrt{x^3 - 8x^2} \, dx \)

(II) What are the units for the integral above? \( \text{Answer: (d)} \)

(a) Lbs  
(b) Newtons  
(c) Kilograms  
(d) Ft-Lbs.  
(e) Joules  
(f) None of these.

20. A cylindrical tank lying on its side has diameter 8 meters and length 12 meters. It is filled with water to a depth of 3 meters. The variable \( x \) measures distance from the bottom of the tank. The density of water is 9800 N/m\(^3\).

(I) Which integral gives the work necessary to pump the water out of the tank? \( \text{Answer: (a)} \)

(a) \( 235200 \int_0^3 (8-x) \sqrt{8x - x^2} \, dx \)  
(b) \( 235200 \int_0^8 (8-x) \sqrt{8x - x^2} \, dx \)  
(c) \( 117600 \int_0^3 (10-x) \sqrt{10x - x^2} \, dx \)  
(d) \( 117600 \int_0^3 (8-x) \sqrt{(4-x)^2 - 16} \, dx \)  
(e) \( 235200 \int_0^8 (3-x) \sqrt{3x - x^2} \, dx \)  
(f) \( 117600 \int_0^3 (x-8) \sqrt{x^2 - 8x} \, dx \)  
(g) \( 235200 \int_0^3 (4-x) \sqrt{4x - x^2} \, dx \)

(II) What are the units for the integral above? \( \text{Answer: (e)} \)

(a) Lbs  
(b) Newtons  
(c) Kilograms  
(d) Ft-Lbs.  
(e) Joules  
(f) None of these.

21. A 4 kg mass is hanging on a 30 meter rope as shown. The rope has a mass of 15 kgs. Which integral gives the work done in lifting the rope 10 meters? (Note that \( x \) measures the length of the rope.) \( \text{Answer: (iii)} \)
\[ \int_{0}^{10} 4.9x + 4 \, dx \quad \text{(i)} \quad \int_{0}^{10} 4.9x + 39.2 \, dx \quad \text{(ii)} \quad \int_{20}^{30} 4.9x + 39.2 \, dx \quad \text{(iii)} \quad \int_{20}^{30} 4.9x + 19.6 \, dx \quad \text{(iv)} \]
\[ \int_{20}^{30} 0.5x^2 + 2 \, dx \quad \text{(v)} \quad \int_{0}^{10} 19.6x + 39.2 \, dx \quad \text{(vi)} \quad \int_{20}^{30} 4.9x + 39.2 \, dx \quad \text{(vii)} \quad \int_{0}^{10} 0.5x + 2 \, dx \quad \text{(viii)} \]

22. What are the units for the answer above? \textbf{Answer: (e) Joules} \begin{itemize}
    \item (a) Lbs
    \item (b) Newtons
    \item (c) Kilograms
    \item (d) Ft-Lbs.
    \item (e) Joules
    \item (f) None of these.
\end{itemize}

23. A 4 lb weight is hanging on a 30 foot rope as shown. The rope weighs 15 lbs. What is the work done in lifting the weight 10 feet? (Note that \( x \) measures the length of the rope.)

\begin{itemize}
    \item (a) 165
    \item (b) 115
    \item (c) 65
    \item (d) 195
    \item (e) 291
    \item (f) 125
\end{itemize}

\textbf{Answer: (a)}

24. What are the units for the answer above? \textbf{Answer: (d)} \begin{itemize}
    \item (a) Lbs
    \item (b) Newtons
    \item (c) Kilograms
    \item (d) Ft-Lbs.
    \item (e) Joules
    \item (f) None of these.
\end{itemize}

25. Which of the following integrals gives the length of the curve \( y = \cos(3x^2) \) from \( x = 0 \) to \( x = 2 \)? \textbf{Answer: (h)} \textbf{None of these. Should be} \[ \int_{0}^{2} \sqrt{1 + 36x^2 \sin^2(3x^2)} \, dx \]
\begin{itemize}
    \item (a) \[ \int_{0}^{2} \sqrt{1 + 6 \sin^2(3x^2)} \, dx \]
    \item (b) \[ \int_{0}^{2} \sqrt{1 + \sin^2(6x)} \, dx \]
    \item (c) \[ \int_{0}^{2} \sqrt{1 + 36 \sin^2(3x^2)} \, dx \]
    \item (d) \[ \int_{0}^{2} \sqrt{1 + 36 \sin^2(6x)} \, dx \]
    \item (e) \[ \int_{0}^{2} \sqrt{1 + 36x^2 \cos^2(3x^2)} \, dx \]
    \item (f) \[ \int_{0}^{2} \sqrt{1 + 6 \sin^2(36x^2)} \, dx \]
    \item (g) \[ \int_{0}^{2} \sqrt{1 + \cos^2(3x^2)} \, dx \]
    \item (h) None of these.
\end{itemize}

26. Which of the following integrals gives the length of the curve \( y = e^{x^2} \) from \( x = 0 \) to \( x = 1 \)? \textbf{Answer: (b)} \begin{itemize}
    \item (a) \[ \int_{0}^{1} \sqrt{1 + e^{x^2}} \, dx \]
    \item (b) \[ \int_{0}^{1} \sqrt{1 + 4x^2 e^{2x^2}} \, dx \]
    \item (c) \[ \int_{0}^{1} \sqrt{1 + x^2 e^{x^2 - 1}} \, dx \]
    \item (d) \[ \int_{0}^{1} \sqrt{1 + (x^2 e^{x^2 - 1})^2} \, dx \]
    \item (e) \[ \int_{0}^{1} \sqrt{1 + e^{2x^2}} \, dx \]
    \item (f) \[ \int_{0}^{1} \sqrt{1 + 2xe^{2x^2}} \, dx \]
    \item (g) \[ \int_{0}^{1} \sqrt{1 + (e^{x^2})^2} \, dx \]
    \item (h) None of these.
\end{itemize}

27. Which of the following integrals gives the length of the curve \( x = t^2, \ y = t^3 \) from \( t = 0 \) to \( t = 1 \)? \textbf{Answer: (a)} \begin{itemize}
    \item (a) \[ \int_{0}^{1} \sqrt{1 + 9t^2} \, dt \]
    \item (b) \[ \int_{0}^{1} \sqrt{4t^2 + 9t^3} \, dt \]
    \item (c) \[ \int_{0}^{1} \sqrt{t^4 + t^5} \, dt \]
    \item (d) \[ \int_{0}^{1} \sqrt{2t + 3t^2} \, dt \]
    \item (e) \[ \int_{0}^{1} \sqrt{1 + 9t^4} \, dt \]
    \item (f) \[ \int_{0}^{1} \sqrt{t^2 + 3t} \, dt \]
    \item (g) \[ \int_{0}^{1} \sqrt{t^3 / 3 + t^4 / 4} \, dt \]
    \item (h) None of these.
\end{itemize}