1. To find \( \int x^2 \sqrt{16 - x^2} \, dx \) by trig substitution, the first step you should make is to let:

(a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)  
(b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)  
(c) \( x = 4 \tan \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)  
(e) \( x = 4 \sec \theta \) and \( dx = 4 \sec^2 \theta \tan \theta \, d\theta \)  
(f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(g) none of these

Answer: (d)

2. To find \( \int \frac{x^3}{\sqrt{x^2 - 16}} \, dx \) by trig substitution, the first step you should make is to let:

(a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)  
(b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)  
(c) \( x = \sin \theta \) and \( dx = \cos \theta \, d\theta \)  
(d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)  
(e) \( x = \sec \theta \) and \( dx = \sec \theta \tan \theta \, d\theta \)  
(f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(g) none of these

Answer: (f)

3. To find \( \int \frac{\sqrt{x^2 + 16}}{x^7} \, dx \) by trig substitution, the first step you should make is to let:

(a) \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)  
(b) \( x = 4 \tan \theta \) and \( dx = 4 \sec^2 \theta \, d\theta \)  
(c) \( x = \sin \theta \) and \( dx = \cos \theta \, d\theta \)  
(d) \( x = 4 \sin \theta \) and \( dx = 4 \cos \theta \, d\theta \)  
(e) \( x = \sec \theta \) and \( dx = \sec \theta \tan \theta \, d\theta \)  
(f) \( x = 4 \sec \theta \) and \( dx = 4 \sec \theta \tan \theta \, d\theta \)  
(g) none of these

Answer: (b)

4. To find \( \int x^3 \sqrt{9 - x^2} \, dx \) by trig substitution, we let \( x = 3 \sin \theta \) and \( dx = 3 \cos \theta \, d\theta \). After simplifying, the integral becomes:

(a) \( \int 243 \sin^3 \theta \cos^2 \theta \, d\theta \)  
(b) \( \int 9 \sin^3 \theta \cos \theta \, d\theta \)  
(c) \( \int 27 \sin^2 \theta \cos^2 \theta \, d\theta \)  
(d) \( \int 243 \sin^3 \theta \cos \theta \, d\theta \)  
(e) \( \int 81 \sin^3 \theta \sqrt{9 - \sin^2 \theta} \cos \theta \, d\theta \)  
(f) \( \int 27 \sin^3 \theta \sqrt{9 - 9 \sin^2 \theta} \, d\theta \)  
(g) none of these

Answer: (a)

5. To find \( \int x^2 \sqrt{9 + x^2} \, dx \) by trig substitution, we let \( x = 3 \tan \theta \) and \( dx = 3 \sec^2 \theta \, d\theta \). After simplifying, the integral becomes:

(a) \( \int 243 \tan^2 \theta \sec^2 \theta \, d\theta \)  
(b) \( \int 27 \tan^3 \theta \sec \theta \, d\theta \)  
(c) \( \int 81 \tan^2 \theta \sec^3 \theta \, d\theta \)  
(d) \( \int 81 \sec^3 \theta \tan^3 \theta \, d\theta \)  
(e) \( \int 81 \tan^3 \theta \sqrt{9 + \tan^2 \theta} \sec^2 \theta \, d\theta \)  
(f) \( \int 27 \tan^3 \theta \sqrt{9 + 9 \tan^2 \theta} \, d\theta \)  
(g) none of these

Answer: (c)

6. To find \( \int \cos^6 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:

(a) \( \cos^6 \theta = \cos^2 \theta \cos^2 \theta \cos \theta \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)  
(c) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \)  
(e) \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)  
(g) \( \cos^6 \theta = \cos^2 \theta \cos^2 \theta \cos \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)

(b) \( \sin^8 \theta = \sin^7 \theta \sin \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)  
(d) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)  
(f) \( \sin^2 \theta = 1 - \cos^2 \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)  
(h) \( \sin^8 \theta = \sin^7 \theta \sin \theta \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)

Answer: (c)
7. To find \( \int \cos^7 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:
(a) \( \cos^7 \theta = \cos^6 \theta \cos \theta \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)
(b) \( \sin^8 \theta = \sin^7 \theta \sin \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)
(c) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \)
(d) \( \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \) and \( \sin^2 \theta = 1 - \cos^2 \theta \)
(e) \( \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)
(f) \( \sin^2 \theta = 1 - \cos^2 \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)
(g) \( \cos^7 \theta = \cos^8 \theta \cos \theta \) and \( \cos^2 \theta = 1 - \sin^2 \theta \)
Answer: (g)

8. Find \( \int \cos^3 \theta \sin^8 \theta \, d\theta \)
Answer: \(\frac{1}{9} \sin^9 \theta - \frac{1}{11} \sin^{11} \theta + C\)

9. To find \( \int \sec^4 \theta \tan^4 \theta \, d\theta \) you should transform this integral into:
(a) \( \int \sec^2 \theta \tan^4 \theta \, d\theta = \int \sec^2 \theta (\sec^2 \theta - 1) \frac{\tan^4 \theta}{\sec^2 \theta} \, d\theta = \int \sec^8 \theta - 2 \sec^6 \theta + \sec^4 \theta \, d\theta \)
(b) \( \int \sec^2 \theta \tan^4 \theta \, d\theta = \int (1 + \tan^2 \theta) \tan^4 \theta \, d\theta = \int (\tan^4 \theta + \tan^6 \theta) \, d\theta \)
(c) \( \int \sec^3 \theta \tan \theta \, d\theta = \int \sec^2 \theta \sec \theta \tan \theta \, d\theta = \int \sec^2 \theta (\sec \theta \tan \theta)^2 \, d\theta \)
(d) \( \int \sec^2 \theta \tan^2 \theta \, d\theta = \int (1 + \tan^2 \theta) (\sec^2 \theta - 1) \sec \theta \tan \theta \, d\theta \)
(f) \( \int (1 + \tan^2 \theta)^2 (\sec^2 \theta - 1)^2 \, d\theta \)
Answer: (b)

10. Find \( \int \sec^4 \theta \tan^4 \theta \, d\theta \).
Answer: \(\frac{1}{5} \tan^5 \theta + \frac{1}{7} \tan^7 \theta + C\)

11. Find the correct standard form of the partial fractions decomposition for \( \frac{x^3 + x + 200}{(x^2 + x - 2)(x - 1)^2(x^2 + 4)^2} \). Do not find the constants.
(a) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2} \)
(b) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{D}{(x - 1)^3} + \frac{Ex}{x^2 + 4} + \frac{Fx}{(x^2 + 4)^2} \)
(c) \( \frac{Ax + B}{x^2 + x - 2} + \frac{C}{x - 1} + \frac{D}{(x - 1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2} \)
(d) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} + \frac{Dx}{x^2 + 4} + \frac{Ex}{(x^2 + 4)^2} \)
(e) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{Cx + D}{(x - 1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2} \)
(f) \( \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{Cx + D}{(x - 1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{G}{x^2 + 4} + \frac{H}{(x^2 + 4)^2} \)
Answer: (a)

12. In the partial fraction expansion \( \frac{x}{(x - 1)(x - 4)^2(x + 2)} = \frac{A}{x - 1} + \frac{B}{(x - 4)^2} + \frac{C}{x - 4} + \frac{D}{x + 2} \), the value of \( A \) is
(a) \( \frac{1}{81} \)  
(b) \( \frac{1}{3} \)
(c) \( \frac{1}{9} \)
(d) \( \frac{1}{9} \)
(e) \( \frac{2}{9} \)
(f) none of these
Answer: (f) it is \( \frac{1}{27} \)
13. Given that the partial fractions expansion of \( \frac{x^3}{(x - 1)^2(x^2 + 1)} \) is \( \frac{1}{2} \cdot \frac{1}{x^2 + 1} - \frac{1}{2} \cdot \frac{1}{x - 1} - \frac{1}{(x - 1)^2} \), find \( \int \frac{x^3}{(x - 1)^2(x^2 + 1)} \, dx \)

Answer: \( -\frac{1}{2(x - 1)} + \ln|x - 1| - \frac{1}{2} \tan^{-1}(x) + C \)

14. (A) Find the partial fractions expansion of \( \frac{x + 7}{x^2 - x - 2} \).

Answer: \( \frac{3}{x - 2} \cdot \frac{2}{x + 1} \)

(B) Evaluate \( \int \frac{x + 7}{x^2 - x - 2} \, dx \) using partial fractions.

Answer: \( 3 \ln|x - 2| - 2 \ln|x + 1| \)

15. To find \( \int \frac{2x + 1}{\sqrt{x^2 + x}} \, dx \), the appropriate method you should use is:

(A) substitution with \( u = x^2 + x \) and \( du = (2x + 1) \, dx \)

(B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)

(C) trig substitution with \( x = \) \( dx = \)

(D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

Answer: \( 2 \sqrt{x^2 + x} + C \)

16. To find \( \int \frac{y + 1}{\sqrt{y}} \, dy \), the appropriate method you should use is:

(A) substitution with \( u = \) \( du = \)

(B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)

(C) trig substitution with \( x = \) \( dx = \)

(D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

Answer: \( 2y^{\frac{3}{2}} + \frac{2}{3}y^{\frac{3}{2}} + C \)

17. To find \( \int x^6 \sqrt{x^2 + 1} \, dx \), the appropriate method you should use is:

(A) substitution with \( u = \) \( du = \)

(B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)

(C) trig substitution with \( x = \tan \theta \) and \( dx = \sec^2 \theta \, d\theta \)

(D) None of the three methods. It can be done directly without them.

**Do not evaluate the integral**

18. To find \( \int \frac{t^3}{t^4 + 11} \, dt \) the appropriate method you should use is:

(A) substitution with \( u = t^4 + 11 \) and \( du = 4t^3 \, dt \)

(B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)

(C) trig substitution with \( t = \) \( dt = \)

(D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

Answer: \( \frac{1}{4} \ln(t^4 + 11) + C \)

19. To find \( \int \frac{s^4 + 11}{s^3} \, ds \) the appropriate method you should use is:

(A) substitution with \( u = \) \( du = \)

(B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)

(C) trig substitution with \( s = \) \( ds = \)

(D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

Answer: \( \frac{1}{3} s^2 - \frac{11}{2} s^{-2} + C \)

20. To find \( \int \frac{x^8}{\sqrt{x^2 - 1}} \, dx \), the appropriate method you should use is:

(A) substitution with \( u = \) \( du = \)
(B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)
(C) trig substitution with \( x = \sec \theta \) and \( dx = \sec \theta \tan \theta \, d\theta \)
(D) None of the three methods. It can be done directly without them.

**Do not evaluate the integral.**

21. To find \( \int \ln z \, dz \) the appropriate method you should use is:
   (A) substitution with \( u = \ldots \) and \( du = \ldots \)
   (B) parts with \( u = \ln z \), \( dv = \frac{1}{z} \, dz \) and \( v = z \)
   (C) trig substitution with \( z = \ldots \) and \( dz = \ldots \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

Answer: \( z \ln (z) - z + C \)

22. To find \( \int_0^2 xe^x \, dx \) the appropriate method you should use is:
   (A) substitution with \( u = \ldots \) and \( du = \ldots \)
   (B) parts with \( u = x \), \( dv = e^x \, dx \) and \( v = e^x \)
   (C) trig substitution with \( x = \ldots \) and \( dx = \ldots \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

Answer: \( xe^x - e^x \bigg|_0^2 = 1 + e^2 \)

23. To find \( \int re^{r^2} \, dr \) the appropriate method you should use is:
   (A) substitution with \( u = r^2 \) and \( du = 2r \, dr \)
   (B) parts with \( u = \ldots \) \( dv = \ldots \) \( du = \ldots \) and \( v = \ldots \)
   (C) trig substitution with \( r = \ldots \) and \( dr = \ldots \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

Answer: \( \frac{1}{2} e^{r^2} + C \)

24. To find \( \int \theta^3 \cos \theta \, d\theta \) the appropriate method you should use is:
   (A) substitution with \( u = \ldots \) and \( du = \ldots \)
   (B) parts with \( u = \theta^3 \), \( dv = \cos \theta \, d\theta \) and \( v = \sin \theta \)
   (C) trig substitution with \( \theta = \ldots \) and \( d\theta = \ldots \)
   (D) None of the three methods. It can be done directly without them.

**Do not evaluate the integral.**

25. Use (a) Simpson’s rule, (b) the trapezoidal rule, (c) a left end point Riemann sum, and (d) a right end point Riemann sum all with 6 subintervals to estimate \( \int_1^4 f(x) \, dx \) using the function values from the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>2.5</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>3.5</td>
<td>2.5</td>
<td>4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Answers: (a) Simpson: 1.833 (b) Trap: 1.5 (c) Left: 1 (d) Right: 2
26. Use (a) Simpson’s rule and (b) the trapezoidal rule both with 4 subintervals to estimate \( \int_{1}^{4} \frac{1}{x} \, dx \).

   Answers: (a) Simpson: 1.39 Trap: 1.43

27. Example 5 Sec 7.6 (page 484).

28. The distances across a lake are 8, 70, 60, 45, 10, and 5 meters as shown. They were measured at 6 meter intervals. Use (I) the Trapezoidal Rule and (II) Simpson’s rule to estimate the area of the lake. **Please show your work.**

   (I) The Trapezoidal Rule gives:
   \[(a) \ 1164 \quad (b) \ 388 \quad (c) \ 594 \quad (d) \ 990 \quad (e) \ 1140 \quad (f) \ 995\]
   Answer: (a) \( \frac{6}{2} \times (8 + 140 + 120 + 90 + 20 + 10 + 0) = 1164 \)

   (II) Simpson’s Rule gives:
   \[(a) \ 628 \quad (b) \ 1980 \quad (c) \ 594 \quad (d) \ 990 \quad (e) \ 1140 \quad (f) \ 1256\]
   Answer: (f) \( \frac{6}{3} \times (8 + 280 + 120 + 180 + 20 + 20 + 0) = 1256 \)

29. Let \( g \) be a function on \([1, 4]\) such that \(-6 \leq g^{(4)}(x) \leq 3\) for \(1 \leq x \leq 4\).

   (I) In the error formula for Simpson’s Rule, what is the best value of \( K \)?
   \[(a) \ 0 \quad (b) \ 4 \quad (c) \ -6 \quad (d) \ 6 \quad (e) \ 5 \quad (f) \ 1 \quad (g) \ none \ of \ these.\]
   Answer: (d)

   (II) What is the smallest number of subintervals one should use in order to assure that Simpson’s Rule approximates \( \int_{1}^{4} g(x) \, dx \) to within \( 10^{-4} \)?
   \[(a) \ 25 \quad (b) \ 13 \quad (c) \ 8100 \quad (d) \ 16 \quad (e) \ 17 \quad (f) \ 18 \quad (g) \ none \ of \ these.\]
   Answer: (f) (remember \( n \) must be even!)

30. Let \( f \) be a function on \([1, 4]\) such that \(-1 \leq f''(x) \leq 5\) for all \( x \) in \([1, 4]\). What is the smallest number of subintervals one should use in order to assure that the Trapezoidal Rule approximates \( \int_{1}^{4} f(x) \, dx \) to within \( 10^{-4} \)?

   \[(a) \ 1125 \quad (b) \ 336 \quad (c) \ 112 \quad (d) \ 11,250 \quad (e) \ 335 \quad (f) \ 1060 \quad (g) \ none \ of \ these.\]
   Answer: (b)