1. To find \( \int x^2 \sqrt{16 - x^2} \, dx \) by trig substitution, the first step you should make is to let:
   \[ (a) \ x = \tan \theta \text{ and } dx = \sec^2 \theta \, d\theta \quad (b) \ x = 4 \tan \theta \text{ and } dx = 4 \sec^2 \theta \, d\theta \quad (c) \ x = 4 \tan \theta \text{ and } dx = 4 \sec \theta \tan \theta \, d\theta \]
   \[ (d) \ x = 4 \sin \theta \text{ and } dx = 4 \cos \theta \, d\theta \quad (e) \ x = 4 \sec \theta \text{ and } dx = 4 \sec^2 \theta \tan \theta \, d\theta \quad (f) \ x = 4 \sec \theta \text{ and } dx = 4 \sec \theta \tan \theta \, d\theta \quad (g) \text{ none of these} \]

2. To find \( \int \frac{x^3}{\sqrt{x^2 - 16}} \, dx \) by trig substitution, the first step you should make is to let:
   \[ (a) \ x = \tan \theta \text{ and } dx = \sec^2 \theta \, d\theta \quad (b) \ x = 4 \tan \theta \text{ and } dx = 4 \sec^2 \theta \, d\theta \quad (c) \ x = \sin \theta \text{ and } dx = \cos \theta \, d\theta \]
   \[ (d) \ x = 4 \sin \theta \text{ and } dx = 4 \cos \theta \, d\theta \quad (e) \ x = \sec \theta \text{ and } dx = \sec \theta \tan \theta \, d\theta \quad (f) \ x = 4 \sec \theta \text{ and } dx = 4 \sec \theta \tan \theta \, d\theta \quad (g) \text{ none of these} \]

3. To find \( \int \frac{\sqrt{x^2 + 16}}{x^3} \, dx \) by trig substitution, the first step you should make is to let:
   \[ (a) \ x = \tan \theta \text{ and } dx = \sec^2 \theta \, d\theta \quad (b) \ x = 4 \tan \theta \text{ and } dx = 4 \sec^2 \theta \, d\theta \quad (c) \ x = \sin \theta \text{ and } dx = \cos \theta \, d\theta \]
   \[ (d) \ x = 4 \sin \theta \text{ and } dx = 4 \cos \theta \, d\theta \quad (e) \ x = \sec \theta \text{ and } dx = \sec \theta \tan \theta \, d\theta \quad (f) \ x = 4 \sec \theta \text{ and } dx = 4 \sec \theta \tan \theta \, d\theta \quad (g) \text{ none of these} \]

4. To find \( \int x^3 \sqrt{9 - x^2} \, dx \) by trig substitution, we let \( x = 3 \sin \theta \) and \( dx = 3 \cos \theta \, d\theta \). After simplifying, the integral becomes:
   \[ (a) \ \int 243 \sin^3 \theta \cos^2 \theta \, d\theta \quad (b) \ \int 9 \sin^3 \theta \cos \theta \, d\theta \quad (c) \ \int 27 \sin^2 \theta \cos^2 \theta \, d\theta \quad (d) \ \int 243 \sin^3 \theta \cos \theta \, d\theta \]
   \[ (e) \ \int 81 \sin^3 \theta \sqrt{9 - 9 \sin^2 \theta} \cos \theta \, d\theta \quad (f) \ \int 27 \sin^3 \theta \sqrt{9 - 9 \sin^2 \theta} \, d\theta \quad (g) \text{ none of these} \]

5. To find \( \int x^2 \sqrt{9 + x^2} \, dx \) by trig substitution, we let \( x = 3 \tan \theta \) and \( dx = 3 \sec^2 \theta \, d\theta \). After simplifying, the integral becomes:
   \[ (a) \ \int 243 \tan^2 \theta \sec^2 \theta \, d\theta \quad (b) \ \int 27 \tan^3 \theta \sec \theta \, d\theta \quad (c) \ \int 81 \tan^2 \theta \sec^3 \theta \, d\theta \quad (d) \ \int 81 \sec^3 \theta \tan^3 \theta \, d\theta \]
   \[ (e) \ \int 81 \tan^3 \theta \sqrt{9 + 9 \tan^2 \theta} \sec^2 \theta \, d\theta \quad (f) \ \int 27 \tan^3 \theta \sqrt{9 + 9 \tan^2 \theta} \, d\theta \quad (g) \text{ none of these} \]

6. To find \( \int \cos^6 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:
   \[ (a) \ \cos^6 \theta = \cos^2 \theta \cos^2 \theta \text{ and } \sin^2 \theta = 1 - \cos^2 \theta \quad (b) \ \sin^8 \theta = \sin^4 \theta \sin^4 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \]
   \[ (c) \ \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \text{ and } \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \quad (d) \ \cos^4 \theta = \frac{1 + \cos(2\theta)}{2} \text{ and } \sin^2 \theta = 1 - \cos^2 \theta \]
   \[ (e) \ \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \quad (f) \ \sin^6 \theta = 1 - \cos^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \]
   \[ (g) \ \cos^6 \theta = \cos^2 \theta \cos^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \quad (h) \ \sin^8 \theta = \sin^4 \theta \sin^4 \theta \text{ and } \sin^2 \theta = 1 - \cos^2 \theta \]

7. To find \( \int \cos^7 \theta \sin^8 \theta \, d\theta \) the first step you should make is to let:
   \[ (a) \ \cos^7 \theta = \cos^6 \theta \cos \theta \text{ and } \sin^2 \theta = 1 - \cos^2 \theta \quad (b) \ \sin^8 \theta = \sin^7 \theta \sin \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \]
   \[ (c) \ \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \text{ and } \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \quad (d) \ \cos^4 \theta = \frac{1 + \cos(2\theta)}{2} \text{ and } \sin^2 \theta = 1 - \cos^2 \theta \]
   \[ (e) \ \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \quad (f) \ \sin^6 \theta = 1 - \cos^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \]
   \[ (g) \ \cos^6 \theta = \cos^2 \theta \cos^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \quad (h) \ \sin^8 \theta = \sin^7 \theta \sin \theta \text{ and } \sin^2 \theta = 1 - \cos^2 \theta \]

8. Find \( \int \cos^3 \theta \sin^8 \theta \, d\theta \)
9. To find \( \int \sec^4 \theta \tan^4 \theta \, d\theta \) you should transform this integral into:

(a) \( \int \sec^2 \theta \tan^3 \theta \sec^2 \theta \, d\theta = \int \sec^2 \theta (\sec^2 \theta - 1)^2 \sec^2 \theta \, d\theta = \int \sec^8 \theta - 2 \sec^6 \theta + \sec^4 \theta \; d\theta \)

(b) \( \int \sec^2 \theta \tan^4 \theta \sec^2 \theta \, d\theta = \int (1 + \tan^2 \theta) \tan^4 \theta \sec^2 \theta \, d\theta = \int (\tan^4 \theta + \tan^6 \theta) \sec^2 \theta \, d\theta \)

(c) \( \int \sec^3 \theta \tan^3 \theta \sec \theta \tan \theta \, d\theta = \int \sec^2 \theta \sec \theta \tan^2 \theta \sec \theta \sec \theta \tan \theta \, d\theta = \int \tan^2 \theta (\sec \theta \tan \theta)^2 \sec^2 \theta \, d\theta \)

(d) \( \int \sec^2 \theta \tan^4 \theta \sec^2 \theta \, d\theta = \int (1 + \tan^2 \theta)(\sec^2 \theta - 1) \sec^2 \theta \, d\theta \)

(e) \( \int \sec^2 \theta \tan^2 \sec^2 \theta \, d\theta = \int (1 + \tan^2 \theta)(\sec^2 \theta - 1) \sec \theta \tan \theta \, d\theta \)

(f) \( \int (1 + \tan^2 \theta)^2 (\sec^2 \theta - 1); \, d\theta \)

10. Find \( \int \sec^4 \theta \tan \theta \, d\theta \).

11. Find the correct standard form of the partial fractions decomposition for \( \frac{x^3 + x + 200}{(x^2 + x - 2)(x - 1)^2(x^2 + 4)^2} \). Do not find the constants.

(a) \( \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2} \)

(b) \( \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3} + \frac{Ex}{x^2 + 4} + \frac{F}{(x^2 + 4)^2} \)

(c) \( \frac{Ax + B}{x^2 + x - 2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{Ex + F}{x^2 + 4} + \frac{Gx + H}{(x^2 + 4)^2} \)

(d) \( \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3} + \frac{Ex}{x^2 + 4} + \frac{F}{(x^2 + 4)^2} \)

(e) \( \frac{A}{x+2} + \frac{B}{x-1} + \frac{Cx + D}{(x-1)^2} + \frac{Ex + F}{(x-1)^3} + \frac{Gx + H}{x^2 + 4} + \frac{H}{(x^2 + 4)^2} \)

(f) \( \frac{A}{x+2} + \frac{B}{x-1} + \frac{Cx + D}{(x-1)^2} + \frac{Ex + F}{(x-1)^3} + \frac{G}{x^2 + 4} + \frac{H}{(x^2 + 4)^2} \)

12. In the partial fraction expansion \( \frac{x}{(x-1)(x-2)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-4)^2} + \frac{C}{x-4} + \frac{D}{x+2} \), the value of \( A \) is

(a) 1/81  
(b) 1/3  
(c) -1/3  
(d) 1/9  
(e) 2/9  
(f) none of these

13. Given that the partial fractions expansion of \( \frac{x^3}{(x-1)(x^2+1)} \) is \( \frac{1}{7} \frac{1}{(x-1)^2} + \frac{1}{x-1} - \frac{1}{7} \frac{1}{x^2+1} \), find \( \int \frac{x^3}{(x-1)^2(x^2+1)} \, dx \)

14. (A) Find the partial fractions expansion of \( \frac{x + 7}{x^2 - x - 2} \).

(B) Evaluate \( \int \frac{x + 7}{x^2 - x - 2} \, dx \) using partial fractions.

15. To find \( \int \frac{2x + 1}{\sqrt{x^2 + x}} \, dx \), the appropriate method you should use is:

(A) substitution with \( u = \) and \( du = \)

(B) parts with \( u = \) \( dv = \) and \( du = \) \( v = \)

(C) trig substitution with \( x = \) and \( dx = \)

(D) None of the three methods. It can be done directly without them.

Evaluate the integral and show your work.
16. To find \( \int \frac{y + 1}{\sqrt{y}} \, dy \), the appropriate method you should use is:
   (A) substitution with \( u = \) and \( du = \)
   (B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)
   (C) trig substitution with \( x = \) \( dx = \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

17. To find \( \int x^6 \sqrt{x^2 + 1} \, dx \), the appropriate method you should use is:
   (A) substitution with \( u = \) and \( du = \)
   (B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)
   (C) trig substitution with \( x = \) \( dx = \)
   (D) None of the three methods. It can be done directly without them.

**Do not evaluate the integral**

18. To find \( \int \frac{t^3}{t^4 + 11} \, dt \) the appropriate method you should use is:
   (A) substitution with \( u = \) and \( du = \)
   (B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)
   (C) trig substitution with \( t = \) \( dt = \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

19. To find \( \int \frac{s^4 + 11}{s^3} \, ds \) the appropriate method you should use is:
   (A) substitution with \( u = \) and \( du = \)
   (B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)
   (C) trig substitution with \( s = \) \( ds = \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

20. To find \( \int \frac{x^8}{\sqrt{x^2 - 1}} \, dx \), the appropriate method you should use is:
   (A) substitution with \( u = \) and \( du = \)
   (B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)
   (C) trig substitution with \( x = \) \( dx = \)
   (D) None of the three methods. It can be done directly without them.

**Do not evaluate the integral.**

21. To find \( \int \ln z \, dz \) the appropriate method you should use is:
   (A) substitution with \( u = \) and \( du = \)
   (B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)
   (C) trig substitution with \( z = \) \( dz = \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

22. To find \( \int_0^2 xe^x \, dx \) the appropriate method you should use is:
   (A) substitution with \( u = \) and \( du = \)
   (B) parts with \( u = \) \( dv = \) \( du = \) \( v = \)
   (C) trig substitution with \( x = \) \( dx = \)
   (D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**
23. To find \( \int r e^r \, dr \) the appropriate method you should use is:

(A) substitution with \( u = \) \( \) and \( du = \) \( \)
(B) parts with \( u = \) \( \) \( dv = \) \( \) \( du = \) \( \) \( \) and \( v = \) \( \)
(C) trig substitution with \( r = \) \( \) \( dr = \) \( \)
(D) None of the three methods. It can be done directly without them.

**Evaluate the integral and show your work**

24. To find \( \int \theta^3 \cos \theta \, d\theta \) the appropriate method you should use is:

(A) substitution with \( u = \) \( \) and \( du = \) \( \)
(B) parts with \( u = \) \( \) \( dv = \) \( \) \( du = \) \( \) \( \) and \( v = \) \( \)
(C) trig substitution with \( \theta = \) \( \) \( d\theta = \) \( \)
(D) None of the three methods. It can be done directly without them.

**Do not evaluate the integral.**

25. Use (a) Simpson’s rule, (b) the trapezoidal rule, (c) a left end point Riemann sum, and (d) a right end point Riemann sum all with 6 subintervals to estimate \( \int_1^4 f(x) \, dx \) using the function values from the table.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2.5</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>3.5</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

26. Use (a) Simpson’s rule and (b) the trapezoidal rule with 4 subintervals to estimate \( \int_1^4 \frac{1}{x} \, dx \).

27. Example 5 Sec 7.6 (page 484).

28. The distances across a lake are 8, 70, 60, 45, 10, and 5 meters as shown. They were measured at 6 meter intervals. Use (I) the Trapezoidal Rule and (II) Simpson’s rule to estimate the area of the lake.

(I) The Trapezoidal Rule gives:
(a) 1164 (b) 388 (c) 594 (d) 990 (e) 1140 (f) 995

(II) Simpson’s Rule gives:
(a) 628 (b) 1980 (c) 594 (d) 990 (e) 1140 (f) 1256

29. Let \( g \) be a function on \([1, 4]\) such that \(-6 \leq g^{(4)}(x) \leq 3\) for \(1 \leq x \leq 4\).

(I) In the error formula for Simpson’s Rule, what is the best value of \( K \)?

(a) 0 (b) 4 (c) -6 (d) 6 (e) 5 (f) 1 (g) none of these.

(II) What is the smallest number of subintervals one should use in order to assure that Simpson’s Rule approximates \( \int_1^4 g(x) \, dx \) to within \( 10^{-4} \)?

(a) 25 (b) 13 (c) 8100 (d) 16 (e) 17 (f) 18 (g) none of these.

30. Let \( f \) be a function on \([1, 4]\) such that \(-1 \leq f''(x) \leq 5\) for all \( x \) in \([1, 4]\). What is the smallest number of subintervals one should use in order to assure that the Trapezoidal Rule approximates \( \int_1^4 f(x) \, dx \) to within \( 10^{-4} \)?

(a) 1125 (b) 336 (c) 112 (d) 11,250 (e) 335 (f) 1060 (g) none of these.