1. Which of the following is logically equivalent to the negation of \( \exists r \ni (r < 1 \text{ AND } r^2 > 1) \)? (There are TWO correct answers.)

(a) \( \forall r \ (r^2 \leq 1 \text{ OR } r \geq 1) \)  
(b) \( \exists r \ni (r \geq 1 \text{ AND } r^2 \geq 1) \)  
(c) \( \forall r \ (r < 1 \Rightarrow r^2 < 1) \)  
(d) \( \forall r \ (r < 1 \Rightarrow r^2 \leq 1) \)  
(e) \( \forall r \ (r \geq 1 \text{ OR } r^2 < 1) \)  
(f) \( \exists r \ni (r < 1 \text{ OR } r^2 > 1) \)

2. Which of the following is logically equivalent to the negation of \( \forall r \exists t \ni (x < t \Rightarrow y < r) \)? (There is only one correct answer.)

(a) \( \forall r \exists t \ni (x \geq t \Rightarrow y \geq r) \)  
(b) \( \exists r \exists t \ni \forall x (x < t \text{ AND } y \geq r) \)  
(c) \( \exists r \ni \forall t (x \geq t \text{ AND } y \geq r) \)  
(d) \( \forall r \exists t (x < t \text{ OR } y \geq r) \)  
(e) \( \exists r \ni \forall t (x \geq t \text{ AND } y < r) \)  
(f) \( \forall r \exists t (x < t \text{ AND } y \geq r) \)  
(g) \( \forall r \ni \exists t (y \geq r \Rightarrow x \geq t) \)  
(h) none of these.

3. Which of the following is logically equivalent to the negation of \( \forall r \exists t \ni (x < t \Rightarrow y < r) \)? (There is only one correct answer.)

(a) \( \forall r \exists t \ni (x \geq t \Rightarrow y \geq r) \)  
(b) \( \exists r \ni \forall t (x < t \text{ AND } y \geq r) \)  
(c) \( \exists r \ni \forall t (x \geq t \text{ AND } y \geq r) \)  
(d) \( \exists r \ni \forall t (x < t \text{ OR } y \geq r) \)  
(e) \( \exists r \ni \forall t (x \geq t \text{ AND } y < r) \)  
(f) \( \forall r \exists t (x < t \text{ AND } y \geq r) \)  
(g) \( \exists r \ni \forall t(y \geq r \Rightarrow x \geq t) \)  
(h) none of these.

4. Solve \( x + \sqrt{2 - x} = 0 \). (This means to prove your answer is correct.)

5. Prove that if \( x \) is an irrational number and \( r \) is a rational number then \( x + r \) is irrational.

6. Let \( f : \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = 3x + 2 \).

(a) Prove that \( f \) is 1-1.

(b) Prove that \( f \) is onto.

(c) Find \( f^{-1}(x) \).

7. Let \( f : \mathbb{N} \to \mathbb{N} \) be defined by \( f(x) = 3x + 2 \). Prove that \( f \) is not onto.

8. Let \( f : A \to B \) and \( g : B \to C \) both be onto. Prove \( g \circ f \) is onto.

9. Let \( f : A \to B \) and \( g : B \to C \). Prove that if \( g \circ f \) is 1-1 then \( f \) is 1-1.

10. Use the Archimedean property to prove that \( \text{inf}(S) = 0 \) where \( S = \left\{ \frac{2}{n+1} \mid n \in \mathbb{N} \right\} \).
11. Use the Archimedean property to prove that \( \frac{n}{2n+1} \to \frac{1}{2} \).

12. Let \( S = \{ x \in \mathbb{R} | x < 3 \} \) prove that \( \text{sup}(S) = 3 \).

13. Let \( x \) be a positive real number. Use mathematical induction to prove the following inequality: 
\[ 1 + nx \leq (1 + x)^n \] for every \( n \in \mathbb{N} \).

14. Use mathematical induction to prove that 
\[ \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \] \( \forall n \in \mathbb{N} \).

15. Prove that if a sequence is convergent then it is bounded. Hint: Use the fact that a sequence \( s_n \) is bounded if and only if \( \exists M \ni \forall n \in \mathbb{N}, |s_n| \leq M \).

16. Prove that \( x_n \to c \) if and only if \( (x_n - c) \to 0 \).

17. Prove that if \( x_n \to a \) and \( y_n \to b \) then \( x_n + y_n \to a + b \).

18. Prove that if \( x_n \to 0 \) and \( \{y_n\} \) is bounded then \( x_n y_n \to 0 \). Hint: Use the fact that a sequence \( s_n \) is bounded if and only if \( \exists M \ni \forall n \in \mathbb{N}, |s_n| \leq M \).

19. Prove that if a sequence converges to \( b \) then every subsequence converges to \( b \).

20. Assume \( f : A \to B \) is a function. **Circle true or false.**

(a) **T** **F** If for each \( a \in A \) there is a \( b \in B \) such that \( f(a) = b \) then \( f \) is onto.

(b) **T** **F** If \( f(x_1) \neq f(x_2) \) whenever \( x_1 \neq x_2 \) then \( f \) is 1-1.

(c) **T** **F** If \( A \) is a bounded subset of \( \mathbb{R} \) then \( \inf(A) \) is the smallest element in \( A \).

(d) **T** **F** If \( A \subseteq \mathbb{R} \) then \( v = \inf(A) \) if and only if \( v \) is a lower bound of \( A \) and given \( r \in \mathbb{R} \) with \( r > v \) then there is a number \( x \in A \) with \( x < r \).

(e) **T** **F** If \( A \subseteq \mathbb{R} \) then \( v = \inf(A) \) if and only if \( v \leq a \forall a \in A \) and given \( \epsilon > 0 \) there is a number \( x \in A \) with \( x > v + \epsilon \).

(f) **T** **F** If \( A \subseteq \mathbb{R} \) then \( u = \sup(A) \) if and only if \( u \geq a \forall a \in A \) and given \( r \in \mathbb{R} \) with \( r > u \) then there is a number \( x \in A \) with \( x < r \).

(g) **T** **F** If \( A \subseteq \mathbb{R} \) then \( u = \sup(A) \) if and only if \( u \geq a \forall a \in A \) and given \( \epsilon > 0 \) there is a number \( x \in A \) with \( x > u - \epsilon \).

(h) **T** **F** If \( x_n \to a \) and \( \{y_n\} \) is bounded then \( x_n y_n \to a \).

(i) **T** **F** “Sup” or “Supremum” means the same thing as “greatest lower bound”.

(j) **T** **F** Between any two real numbers there is always an irrational number.
(k) T F The proof that between any two real numbers there is always a rational number depends on the completeness property of $\mathbb{R}$. 