Math 373 Exam 2 Supplement for Review. This is only a supplement. All problems like those on the homework and quizzes may appear on the exam. Remember - it’s not whether something is true or false, it’s how you write the proof that’s important.

1. Write the domain of \( H \) in interval notation, if \( H(x) = \frac{\sqrt{x^2 - 4}}{x + 11} \).

2. Let \( L : \mathbb{Z} \rightarrow \mathbb{Z} \) by \( L(x) = 1 - 2x \). Is \( L \) surjective? Prove your answer.

3. What is the domain of \( L \)?

4. Let \( s : \mathbb{R} \rightarrow \mathbb{R}^+ \) by \( s(x) = x^2 \). Is \( s \) surjective? Prove your answer.

5. What is the domain of \( s \)?

6. Let \( p : \mathbb{R} \rightarrow \mathbb{R}^+ \) by \( p(x) = x^2 \). Is \( p \) injective? Prove your answer.

7. What is the domain of \( p \)?

8. Let \( G : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) by \( G(x) = x^2 \). Is \( G \) injective? Prove your answer.

9. What is the domain of \( G \)?

10. Let \( f(x) = 1 - 5x \) and \( g(x) = 3x + 1 \). Find and simplify: 
    \((f \circ g)(x), (f \circ f)(x), (g \circ f)(x), \) and \((g \circ g)(x)\).

11. Complete this definition: The sequence \( \{x_n\} \) converges to \( x \) if and only if

12. Write the negation of the above definition using logical symbols:

13. Write the negation of \( \exists r \in \mathbb{R} \ \exists s \in \mathbb{R} \ rs > 0 \) using logical symbols.

14. Which of the above is true, the statement or its negation? Prove your answer.

15. Write the negation of \( \forall r \in \mathbb{R} \ \exists s \in \mathbb{R} \ \exists s > 0 \) using logical symbols.

16. Which of the above is true, the statement or its negation? Prove your answer.

17. Write the negation of \( \forall r \in \mathbb{R} \ \exists s \in \mathbb{Z} \ \exists |r - s| < 2 \) using logical symbols.

18. Which of the above is true, the statement or its negation? (Proof not necessary.)

19. Write the negation of \( \exists s \in \mathbb{Z} \ \forall r \in \mathbb{R} \ |r - s| < 2 \) using logical symbols.

20. Which of the above is true, the statement or its negation? (Proof not necessary.)

21. Prove that \( A \cap \bigcup_{j=1}^{\infty} S_j = \bigcup_{j=1}^{\infty} A \cap S_j \).
22. Let \( A = \{x | x^2 < 4\} \) and let \( B = \{x | x < 2\} \). Is \( A \subseteq B \)? Prove your answer. Is \( B \subseteq A \)? Prove your answer.

23. Let \( f : \mathbb{R} \to \mathbb{R} \) by \( f(x) = x^2 \). Let \( A = (-1, 4) \ B = (-4, -1) \ C = (1, 4) \)

\[
\overrightarrow{f}(A) = \ldots \overrightarrow{f}(A) = \\
\overrightarrow{f}(B) = \ldots \overrightarrow{f}(B) = \\
\overrightarrow{f}(C) = \ldots \overrightarrow{f}(C) = 
\]

24. Let \( f : X \to Y \). Prove that \( \overrightarrow{f}(A \cap B) \subseteq \overrightarrow{f}(A) \cap \overrightarrow{f}(B) \).

25. Let \( f : X \to Y \). Prove that if \( f \) is injective then \( \overrightarrow{f}(A) \cap \overrightarrow{f}(B) \subseteq \overrightarrow{f}(A \cap B) \).

26. Let \( f : X \to Y \) and let \( A_1 \) and \( A_2 \) be subsets of \( Y \).

Prove that \( \overrightarrow{f}(A_1 \cup A_2) \subseteq \overrightarrow{f}(A_1) \cup \overrightarrow{f}(A_2) \).

27. Give an example to show that \( \overrightarrow{f}(A_1) \cap \overrightarrow{f}(A_2) \) need not be a subset of \( \overrightarrow{f}(A_1 \cap A_2) \).

28. Let \( f : X \to Y \) and \( g : Y \to Z \). Prove that if \( g \) and \( f \) are injective then \( g \circ f \) is injective.

29. Let \( f : X \to Y \) and \( g : Y \to Z \). Prove that if \( g \circ f \) is injective then \( f \) is injective.

30. Suppose \( \{a_n\} \) converges to \( a \) and \( c \neq 0 \). Then \( \{ca_n\} \) converges to \( ca \).

31. Suppose \( \{a_n\} \) converges to \( a \) and \( \{b_n\} \) converges to \( b \). Then \( \{a_n - b_n\} \) converges to \( a - b \).

32. Prove that if \( \{x_n\} \) converges to \( x \). Then \( \left\{ \frac{3x_n}{2} \right\} \) converges to \( \frac{3x}{2} \) using only the definition.

33. Prove that \( \left\{ \frac{2n - 3}{n + 1} \right\} \) converges to 2 using only the definition.