The entire process consists of the three subsystems connected in series. Therefore, for the process to function, all three subsystems must function. We conclude that

\[ P(\text{system functions}) = P(\text{systems } 1, 2, \text{ and } 3 \text{ all function}) \]
\[ = p_1 p_2 p_3 \]
\[ = (0.985050)(0.999875)(0.988200) \]
\[ = 0.973 \]

We remark that the assumption that the components function independently is crucial in the solutions of Examples 2.28, 2.29, and 2.30. When this assumption is not met, it can be very difficult to make accurate reliability estimates. If the assumption of independence is used without justification, reliability estimates may be misleading.

Exercises for Section 2.3

1. A box contains 10 fuses. Eight of them are rated at 10 amperes (A) and the other two are rated at 15 A. Two fuses are selected at random.
   a. What is the probability that the first fuse is rated at 15 A?
   b. What is the probability that the second fuse is rated at 15 A, given that the first fuse is rated at 10 A?
   c. What is the probability that the second fuse is rated at 15 A, given that the first fuse is rated at 15 A?

2. Refer to Exercise 1. Fuses are randomly selected from the box, one by one, until a 15 A fuse is selected.
   a. What is the probability that the first two fuses are both 10 A?
   b. What is the probability that a total of two fuses are selected from the box?
   c. What is the probability that more than three fuses are selected from the box?

3. On graduation day at a large university, one graduate is selected at random. Let \( A \) represent the event that the student is an engineering major, and let \( B \) represent the event that the student took a calculus course in college. Which probability is greater, \( P(A|B) \) or \( P(B|A) \)? Explain.

4. The article “Integrating Risk Assessment and Life Cycle Assessment: A Case Study of Insulation” (Y. Nishinaka, J. Levy, et al., Risk Analysis, 2002: 1003–1017) estimates that 5.6% of a certain population has asthma, and that an asthmatic has probability 0.027 of suffering an asthma attack on a given day. A person is chosen at random from this population. What is the probability that this person has an asthma attack on that day?

5. Oil wells drilled in region A have probability 0.2 of producing. Wells drilled in region B have probability 0.09 of producing. One well is drilled in each region. Assume the wells produce independently.
   a. What is the probability that both wells produce?
   b. What is the probability that neither well produces?
   c. What is the probability that at least one of the two produces?

6. Of all failures of a certain type of computer hard drive, it is determined that in 20% of them only the sector containing the file allocation table is damaged, in 70% of them only nonessential sectors are damaged, and in 10% of the cases both the allocation sector and one or more nonessential sectors are damaged. A failed drive is selected at random and examined.
   a. What is the probability that the allocation sector is damaged?
   b. What is the probability that a nonessential sector is damaged?
   c. If the drive is found to have a damaged allocation sector, what is the probability that some nonessential sectors are damaged as well?
   d. If the drive is found to have a damaged nonessential sector, what is the probability that the allocation sector is damaged as well?
c. If the drive is found to have a damaged allocation sector, what is the probability that no nonessential sectors are damaged?

f. If the drive is found to have a damaged nonessential sector, what is the probability that the allocation sector is not damaged?

7. In the process of producing engine valves, the valves are subjected to a first grind. Valves whose thicknesses are within the specification are ready for installation. Those valves whose thicknesses are above the specification are reground, while those whose thicknesses are below the specification are scrapped. Assume that after the first grind, 70% of the valves meet the specification, 20% are reground, and 10% are scrapped. Furthermore, assume that of those valves that are reground, 90% meet the specification, and 10% are scrapped.

a. Find the probability that a valve is ground only once.

b. Given that a valve is not reground, what is the probability that it is scrapped?

c. Find the probability that a valve is scrapped.

d. Given that a valve is scrapped, what is the probability that it was ground twice?

e. Find the probability that the valve meets the specification (after either the first or second grind).

f. Given that a valve meets the specification (after either the first or second grind), what is the probability that it was ground twice?

g. Given that a valve meets the specification, what is the probability that it was ground only once?

8. Sarah and Thomas each roll a die. Whoever gets the higher number wins; if they both roll the same number, neither wins.

a. What is the probability that Thomas wins?

b. If Sarah rolls a 3, what is the probability that she wins?

c. If Sarah rolls a 3, what is the probability that Thomas wins?

d. If Sarah wins, what is the probability that Thomas rolled a 3?

e. If Sarah wins, what is the probability that Sarah rolled a 3?

9. A particular automatic sprinkler system has two different types of activation devices for each sprinkler head. One type has a reliability of 0.9; that is, the probability that it will activate the sprinkler when it should is 0.9. The other type, which operates independently of the first type, has a reliability of 0.8. If either device is triggered, the sprinkler will activate. Suppose a fire starts near a sprinkler head.

a. What is the probability that the sprinkler head will be activated?

b. What is the probability that the sprinkler head will not be activated?

c. What is the probability that both activation devices will work properly?

d. What is the probability that only the device with reliability 0.9 will work properly?

10. Laura and Philip each fire one shot at a target. Laura has probability 0.5 of hitting the target, and Philip has probability 0.3. The shots are independent.

a. Find the probability that the target is hit.

b. Find the probability that the target is hit by exactly one shot.

c. Given that the target was hit by exactly one shot, find the probability that Laura hit the target.

11. A driver encounters two traffic lights on the way to work each morning. Each light is either red, yellow, or green. The probabilities of the various combinations of colors is given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>First Light</th>
<th>Second Light</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>Y</td>
</tr>
<tr>
<td>R</td>
<td>0.30</td>
<td>0.04</td>
</tr>
<tr>
<td>Y</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>G</td>
<td>0.15</td>
<td>0.05</td>
</tr>
</tbody>
</table>

a. What is the probability that the first light is red?

b. What is the probability that the second light is green?

c. Find the probability that both lights are the same color.

d. Given that the first light is red, find the probability that the second light is green.

e. Given that the second light is yellow, find the probability that the first light is red.
12. A fast-food restaurant chain has 600 outlets in the United States. The following table categorizes cities by size and location, and presents the number of restaurants in each category. A restaurant is to be chosen at random from the 600 to test market a new style of chicken.

<table>
<thead>
<tr>
<th>Population of City</th>
<th>Region</th>
<th>NE</th>
<th>SE</th>
<th>SW</th>
<th>NW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 50,000</td>
<td></td>
<td>30</td>
<td>35</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>50,000–500,000</td>
<td></td>
<td>60</td>
<td>90</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>Over 500,000</td>
<td></td>
<td>150</td>
<td>25</td>
<td>30</td>
<td>60</td>
</tr>
</tbody>
</table>

a. If the restaurant is located in a city with a population over 500,000, what is the probability that it is in the Northeast?
b. If the restaurant is located in the Southeast, what is the probability that it is in a city with a population under 50,000?
c. If the restaurant is located in the Southwest, what is the probability that it is in a city with a population of 500,000 or less?
d. If the restaurant is located in a city with a population of 500,000 or less, what is the probability that it is in the Southwest?
e. If the restaurant is located in the South (either SE or SW), what is the probability that it is in a city with a population of 50,000 or more?

13. Nuclear power plants have redundant components in important systems to reduce the chance of catastrophic failure. Assume that a power plant has two gauges to measure the level of coolant in the reactor core and that each gauge has probability 0.01 of failing. Assume that one potential cause of gauge failure is that the electric cables leading from the core to the control room where the gauges are located may burn up in a fire. Someone wishes to estimate the probability that both gauges fail, and makes the following calculation:

\[ P(\text{both gauges fail}) = P(\text{first gauge fails}) \times P(\text{second gauge fails}) = (0.01)(0.01) = 0.0001 \]

a. What assumption is being made in this calculation?
b. Explain why this assumption is probably not justified in the present case.
c. Is the probability of 0.0001 likely to be too high or too low? Explain.

14. Refer to Exercise 13. Is it possible for the probability that both gauges fail to be greater than 0.01? Explain.

15. A lot of 10 components contains 3 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a. Find P(A).
b. Find P(B|A).
c. Find P(A \cap B).
d. Find P(A' \cap B).
e. Find P(B).

16. A lot of 1000 components contains 300 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a. Find P(A).
b. Find P(B|A).
c. Find P(A \cap B).
d. Find P(A' \cap B).
e. Find P(B).
g. Are A and B independent? Is it reasonable to treat A and B as though they were independent? Explain.

17. In a lot of n components, 30% are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective. For which lot size n will A and B be more nearly independent: n = 10 or n = 10,000? Explain.

18. In a Major League Baseball Championship series, the first team to win four games wins the series. In 2004, the Boston Red Sox became the first team in major league history to win four consecutive games after losing the first three, when they defeated the New York...
2.3 Conditional Probability and Independence

Yankees. Suppose that the Red Sox had probability 0.6 of winning each game, and that the games were independent.

a. What is the probability that the Yankees would win the first three games, and the Red Sox the next four?

b. After the Yankees had won the first three games, what was the probability that the Red Sox would win the next four?

19. Once each hour, a manufacturing process is inspected. If a malfunction is detected, the process is shut down. The probability is 0.05 that a malfunction will be detected (falsely) even when the process is functioning properly. Assume the inspections are independent.

a. If the process functions properly for 24 hours, what is the probability that it will be shut down at least once?

b. If \( p \) is the probability of a false detection, what must the value of \( p \) be so that the probability that no malfunctions are falsely detected for 24 hours is 0.80?

20. At a certain car dealership, the probability that a customer purchases an SUV is 0.20, and the probability that a customer purchases a black SUV is 0.05. Given that a customer purchases an SUV, what is the probability that it is black?

21. Each day, a weather forecaster predicts whether or not it will rain. For 80% of rainy days, she correctly predicts that it will rain. For 90% of non-rainy days, she correctly predicts that it will not rain. Suppose that 10% of days are rainy and 90% are non-rainy.

a. What proportion of the forecasts are correct?

b. Another forecaster always predicts that there will be no rain. What proportion of these forecasts are correct?

22. A certain delivery service offers both express and standard delivery. Seventy-five percent of parcels are sent by standard delivery, and 25% are sent by express. Of those sent standard, 80% arrive the next day, and of those sent express, 95% arrive the next day. A record of a parcel delivery is chosen at random from the company’s files.

a. What is the probability that the parcel was shipped express and arrived the next day?

b. What is the probability that it arrived the next day?

c. Given that the package arrived the next day, what is the probability that it was sent express?

23. Electric circuit boards are rated excellent, acceptable, or unacceptable. Suppose that 50% of boards are excellent, 60% are acceptable, and 60% are acceptable. Further, suppose that 10% of excellent boards fail, 20% of acceptable boards fail, and 100% of unacceptable boards fail (unacceptable boards are discarded without being used).

a. What is the probability that a board is rated excellent and fails?

b. What is the probability that a board fails?

c. Given that a board fails, what is the probability that it was rated excellent?

24. Items are inspected for flaws by two quality inspectors. If a flaw is present, it will be detected by the first inspector with probability 0.9, and by the second inspector with probability 0.7. Assume the inspectors function independently.

a. If an item has a flaw, what is the probability that it will be found by both inspectors?

b. If an item has a flaw, what is the probability that it will be found by at least one of the two inspectors?

c. Assume that the second inspector examines only those items that have been passed by the first inspector. If an item has a flaw, what is the probability that the second inspector will find it?

25. Refer to Exercise 24. Assume that both inspectors inspect every item and that if an item has no flaw, then neither inspector will detect a flaw.

a. Assume that the probability that an item has a flaw is 0.10. If an item is passed by the first inspector, what is the probability that it actually has a flaw?

b. Assume that the probability that an item has a flaw is 0.10. If an item is passed by both inspectors, what is the probability that it actually has a flaw?

26. Refer to Example 2.26. Assume that the proportion of people in the community who have the disease is 0.05.

a. Given that the test is positive, what is the probability that the person has the disease?

b. Given that the test is negative, what is the probability that the person does not have the disease?
27. Haemophilia is a sex-linked genetic disease that results in the inability of blood to clot. (A disease is sex-linked if the disease gene is located on the X-chromosome.) A woman with one copy of the gene is a carrier, which means that she does not have the disease, but she can transmit it to her male children. If a carrier has children by a man who is disease-free, each son has probability 0.5 of having the disease, and each daughter has probability 0.5 of being a carrier. Outcomes are independent.

a. A woman who is a carrier has two sons. The father is disease-free. What is the probability that neither of the sons has the disease?

b. A woman whose mother was a carrier has probability 0.5 of being a carrier. If this woman has a son by a man who is disease free, what is the probability that the son has the disease?

c. Refer to part (b). If the son does not have the disease, what is the probability that the woman is a carrier?

28. A quality-control program at a plastic bottle production line involves inspecting finished bottles for flaws such as microscopic holes. The proportion of bottles that actually have such a flaw is only 0.0002. If a bottle has a flaw, the probability is 0.995 that it will fail the inspection. If a bottle does not have a flaw, the probability is 0.999 that it will pass the inspection.

a. If a bottle fails inspection, what is the probability that it has a flaw?

b. Which of the following is the more correct interpretation of the answer to part (a)?
   i. Most bottles that fail inspection do not have a flaw.
   ii. Most bottles that pass inspection do have a flaw.

c. If a bottle passes inspection, what is the probability that it does not have a flaw?

d. Which of the following is the more correct interpretation of the answer to part (c)?
   i. Most bottles that fail inspection do have a flaw.
   ii. Most bottles that pass inspection do not have a flaw.

c. Explain why a small probability in part (e) is not a problem, so long as the probability in part (c) is large.

29. Refer to Example 2.26.
   a. If a man tests negative, what is the probability that he actually has the disease?
   b. For many medical tests, it is standard procedure to repeat the test when a positive signal is given. If repeated tests are independent, what is the probability that a man who tests positive on two successive tests if he has the disease?
   c. Assuming repeated tests are independent, what is the probability that a man tests positive on two successive tests if he does not have the disease?
   d. If a man tests positive on two successive tests, what is the probability that he has the disease?

30. A system consists of four components connected as shown in the following diagram:

   Assume A, B, C, and D function independently. If the probabilities that A, B, C, and D fail are 0.10, 0.05, 0.10, and 0.20, respectively, what is the probability that the system functions?

31. A system consists of four components, connected as shown in the diagram. Suppose that the components function independently, and that the probabilities of failure are 0.05 for A, 0.03 for B, 0.07 for C, and 0.14 for D. Find the probability that the system functions.

32. A system contains two components, A and B, connected in series, as shown in the diagram.

   Assume A and B function independently. For the system to function, both components must function.