Math 373, Study Sheet for Quiz 2, Summer 2010
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(1) State true or false. If true, give a brief justification (not a formal proof) and if false, give a counter example. Let $A$, $B$, $C$, and $D$ be subsets of a universe $U$, not necessarily nonempty.

(a) $A - (B \cup C) = (A - B) \cup (A - C)$.
(b) $A - (B \cup C) = (A - B) \cap (A - C)$.
(c) $A \times B = C \times D \Rightarrow A = C$ and $B = D$.
(d) For nonempty sets $A$, $B$, $C$, and $D$, it is true that $A \times B = C \times D \Rightarrow A = C$ and $B = D$.
(e) $A - B = A - A \cap B$.

(2) Fill in the blanks.

(a) If for $n \in \mathbb{N}$, we have $A_n = [-\frac{1}{n}, \frac{1}{n}] \subset \mathbb{R}$, $B_n = [1 + \frac{1}{n}, n + 2] \subset \mathbb{R}$ and $C_n = [1, n] \subset \mathbb{R}$ then

\[
\bigcap_{i=1}^{10} A_n = \text{_________}, \quad \bigcap_{i=1}^{10} B_n = \text{_________}, \quad \bigcap_{i=1}^{10} C_n = \text{_________},
\]

\[
\bigcap_{i=5}^{10} A_n = \text{_________}, \quad \bigcap_{i=5}^{10} B_n = \text{_________}, \quad \bigcap_{i=5}^{10} C_n = \text{_________},
\]

\[
\bigcup_{i=1}^{\infty} A_n = \text{_________}, \quad \bigcup_{i=1}^{\infty} B_n = \text{_________}, \quad \bigcup_{i=1}^{\infty} C_n = \text{_________},
\]

\[
\bigcup_{i=1}^{10} A_n = \text{_________}, \quad \bigcup_{i=1}^{10} B_n = \text{_________}, \quad \bigcup_{i=1}^{10} C_n = \text{_________}.
\]

(b) If $A$ has $m$ elements and $B$ has $n$ elements, then $A \times B$ has _________ elements.
(c) If $A$ has $m$ elements then the power set $\mathcal{P}(A)$ has _________ elements.
(d) If $A$ has $m$ elements and $B$ has $n$ elements, then there are _________ possible relations from $A$ to $B$.
(e) If $A$ has $m$ elements and $B$ has $n$ elements, then there are _________ possible nonempty relations from $A$ to $B$.
(f) For the set $(2, 3) \subset \mathbb{R}$ the least element is _________, the least upper bound is _________, the greatest element is _________, and the greatest lower bound is _________.
(g) For the set \([-2, 3) \subset \mathbb{R}\) the least element is \(\underline{\text{________}}\), the least upper bound is \(\underline{\text{________}}\), the greatest element is \(\underline{\text{________}}\), and the greatest lower bound is \(\underline{\text{________}}\).

(h) For the set \(\{-5\} \cup (2, 3] \subset \mathbb{R}\) the least element is \(\underline{\text{________}}\), the least upper bound is \(\underline{\text{________}}\), the greatest element is \(\underline{\text{________}}\), and the greatest lower bound is \(\underline{\text{________}}\).

(i) For the set \(\mathbb{N} \subset \mathbb{R}\) the least element is \(\underline{\text{________}}\), the least upper bound is \(\underline{\text{________}}\), the greatest element is \(\underline{\text{________}}\), and the greatest lower bound is \(\underline{\text{________}}\).

(j) For the set \(\{\frac{1}{n} \mid n \in \mathbb{N}\} \subset \mathbb{R}\) the least element is \(\underline{\text{________}}\), the least upper bound is \(\underline{\text{________}}\), the greatest element is \(\underline{\text{________}}\), and the greatest lower bound is \(\underline{\text{________}}\).

(3) Section 4.3, Exercises 1 and 2.