8.2

4. \[ \sum_{n=1}^{\infty} \frac{2n^2-1}{n^2+1} \quad \text{An for large } n \text{ is approximately } \frac{2n^2}{n^2} = 2 \]

so since \( a_n \) does not go to 0, series is divergent.

7. \[ \sum_{n=1}^{\infty} \left[ \frac{1}{n^{1.5}} - \frac{1}{(n+1)^{1.5}} \right] = \left( \frac{1}{1^{1.5}} - \frac{1}{2^{1.5}} \right) + \left( \frac{1}{2^{1.5}} - \frac{1}{3^{1.5}} \right) + \left( \frac{1}{3^{1.5}} - \frac{1}{4^{1.5}} \right) + \ldots \]

\[ = 1 - \lim_{n \to \infty} \frac{1}{(n+1)^{1.5}} = 1 \quad \text{since } \lim_{n \to \infty} \frac{1}{(n+1)^{1.5}} = 0 \]

12. \[ 1 + 0.4 + 0.16 + 0.064 + \ldots = 0.4^0 + 0.4^1 + 0.4^2 + 0.4^3 + \ldots \]

geometric series with \( A = 1 \), \( r = 0.4 \)

sum is \[ \frac{1}{1-0.4} = \frac{1}{0.6} = \frac{5}{3} \]

15. \[ \sum_{n=1}^{\infty} \frac{3^{-n} 8^{n+1}}{n+1} = 8 \sum_{n=1}^{\infty} \left( \frac{8}{3} \right)^n \]

is also geometric but

with \( r = \frac{8}{3} > 1 \) series is divergent.

20. \[ \sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)} \quad A_n \approx \frac{u^2}{u^2} \quad \text{for large } u \]

so \( A_n \to 1 \)

series is divergent.