1. Determine whether each statement is TRUE or FALSE (2 points for correct answer, 1 for no answer, and 0 for wrong answer).

(a) The improper integral \( \int_{1}^{\infty} \frac{1}{x + e^{2x}} \, dx \) is convergent by the comparison test, since \( \frac{1}{x + e^{2x}} < \frac{1}{e^{2x}} \) (this inequality is true).

(b) The \( x \)-coordinate of the center of mass, \( \bar{x} \), of the figure above is greater than 1.

(c) The pdf of the normal distribution with mean 0 and variance 1 is given by 
\[
    f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}
\]
(again, this is true).

Then \( \int_{-\infty}^{\infty} f(x) \, dx = 1 \) (don’t try to integrate).

(d) The arc length of the curve \( y = \frac{x^3}{3} + \frac{1}{x} \) for \( 1 \leq x \leq 3 \) is given by
\[
    L = \int_{1}^{3} \sqrt{x^2 + \left( \ln x \right)^2} \, dx.
\]

2. Show that the integral \( \int_{0}^{1} \frac{1}{x} \, dx \) is divergent by showing that the appropriate limit does not exist.

Look at \( \lim_{t \to 0^+} \int_{t}^{1} \frac{1}{x} \, dx \). Since the antiderivative of \( \frac{1}{x} \) is \( \ln x \),
\[
    \lim_{t \to 0^+} \int_{t}^{1} \frac{1}{x} \, dx = \lim_{t \to 0^+} (\ln 1 - \ln t),
\]
but \( \lim_{t \to 0^+} \ln t \) does not exist (or it is \( -\infty \)). So the integral diverges since it does not have a finite value.
3. Set up an integral to find the area of the region bounded by $y = 4x^2$ and $y = x^2 + 3$, shown below.

$$A = \int_{-1}^{1} (x^2 + 3 - 4x^2) \, dx = 2 \int_{0}^{1} (3 - 3x^2) \, dx$$

4. Set up an integral to find the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = 2$

(a) about the $x$-axis.

$$\text{vol} = \int_{1}^{2} \left( \frac{1}{x} \right)^2 \pi \, dx$$

(b) about the $y$-axis.

$$\text{vol} = \int_{1}^{2} (2 - e^{-y}) \pi \, dy$$

5. The probability that a fair coin lands heads or tails should be $\frac{1}{2}$. However, the probability $X$ that a randomly selected coin (which may not be fair) will land heads can be modeled according to the probability density function (pdf)

$$f(x) = \begin{cases} 6x(1-x) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $f(x)$ is a legitimate pdf. A sketch may help.

$$\int_{0}^{1} 6x(1-x) \, dx = 3 - 2 = 1$$

(b) What is the probability that $X$ is less than $\frac{1}{4}$? Write an integral that represents the probability.

$$\mathbb{P}(X < \frac{1}{4}) = \int_{0}^{\frac{1}{4}} 6x(1-x) \, dx = \left[ 3x^2 - 2x^3 \right]_{0}^{\frac{1}{4}} = \frac{5}{32}$$
6. A force of 30N is required to maintain a spring stretched from its natural length of 12cm to a length of 15cm. How much work is done in stretching the spring from 12cm to 20cm?

\[
\text{force} = k \times x \quad \text{so} \quad 30N = k \times 3 \text{cm.} \quad k = \frac{30}{3} \text{N/cm} = 10 \text{N/cm}
\]

\[
\text{work} = \int_0^8 10x \, dx = 5x^2 \bigg|_0^8 = 320 \text{N.cm or 3.2 N m}
\]

7. The table (supplied by San Diego Gas and Electric) gives the power consumption in gigawatts in San Diego County from midnight to 6:00am on December 8, 1999. It is known that power is the derivative of energy.

<table>
<thead>
<tr>
<th>t (hours after midnight)</th>
<th>P(gigawatts)</th>
<th>t (hours after midnight)</th>
<th>P(gigawatts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.81</td>
<td>4.0</td>
<td>1.62</td>
</tr>
<tr>
<td>1.0</td>
<td>1.69</td>
<td>5.0</td>
<td>1.75</td>
</tr>
<tr>
<td>2.0</td>
<td>1.64</td>
<td>6.0</td>
<td>2.05</td>
</tr>
<tr>
<td>3.0</td>
<td>1.60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Use your choice of approximate integration to estimate the energy used during that time period.

\[
\text{Total energy} = \int_0^6 \text{power} \, dt
\]

\[
\text{left sum} = 1.81 + 1.69 + 1.64 + 1.60 + 1.62 + 1.75 = 10.11 \text{ gigawatts-hour}
\]

(b) Using your answer from part (a), find the average power used between midnight and 6:00am.

\[
\text{av. power} = \frac{1}{6} \int_0^6 \text{power} \, dt
\]

\[
= \frac{10.11}{6} = 1.685 \text{ gigawatts}
\]

If you need to know your updated grade by tomorrow (the last day to drop), please put your e-mail address here______________________________